



## Multi-Frame Super-Resolution Survey

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### ABSTRACT

The image resolution is probably the primary measurements of the image quality. The image with higher resolution is required and generally desired in the majority of applications, because of it represents the additional information inside the image. However, the best using of image sensors and optical technologies is normally a high priced and also restrictive method to increase the image pixel density. Therefore, the effective use of image processing techniques for acquiring a high-resolution image generated from low-resolution images is an inexpensive and a powerful solution. This kind of image improvement is termed super-resolution image. Numerous strategies like frequency domain, interpolation, and regularization techniques are proposed for generating the super resolution image. In this paper, a general survey of the available multi-frame super resolution approaches is explained. Finally, several image quality metrics are discussed to measure the similarity between the reconstructed image and the original image.

**Keywords:** *super resolution, frequency domain, spatial domain, image interpolation, resolution enhancement, regularized framework*

### 1. Introduction

Within the past decade, the global world provides an enormous advancement in software and hardware technologies. The industrial sectors got the benefit of the modern technology to generate electronic devices such as computer systems, cellular mobile phones, personal digital assistant (PDA) and so much more at inexpensive costs. Furthermore, the camera sensor highly developed in their manufacturing methods to generate high-quality digital cameras. Even though, high-resolution (HR) digital cameras are obtainable, many applications of computer vision such as medical imaging, satellite imaging, pattern recognition, surveillance and forensic, astronomical imaging, target detection, and so many

more still got a strong requisition to get HR image which usually frequently surpassed the abilities of the HR digital cameras (Park, Park, & Kang, 2003).

Optical resolution is certainly a method of measuring the capability of the camera system or an element of the camera system to explain the image details. As a result, there are two primary methods of raising the spatial image resolution: a technical strategy that related to hardware solutions and analytical strategy that related to software solutions. The technical strategy concept identifies the registration device improvement or the replacement with a higher resolution device. However, oftentimes the usage of a much better camera is limited simply by its high price, large size or sensor manufacturing limitations. The analytical strategy concept is usually inexpensive and more flexible as compared to the hardware solutions. The class of resolution improvement methods got the name of super-resolution (SR) image reconstruction (Park et al., 2003; Protter, Elad, Takeda, & Milanfar, 2009).

SR image reconstruction is usually a great encouraging method of digital imaging, which tries to rebuild HR images simply by combining the partial information presented inside a several of low-resolution (LR) images of the particular scene through the process of image reconstruction. SR incorporates up-sampling of LR images, and then eliminating distortions such as noising and blurring. Compared to different image improvement techniques, SR not only increases the quality of LR images by improving their particular spatial resolution but also tries to eliminate distortions (Park et al., 2003).

This research paper presents a survey of major multi-frame SR techniques and it is organized as follows. Section 2 explains observation model that relates the HR image to the observed LR images. Several multi-frame SR techniques are described in Section 3. The image quality metrics are discussed in Section 4. A detailed discussion is presented in Section 5, and the paper is concluded in this section.

## 2. Observation Model

The observation model identifies the true way where the observed LR images are acquired. The image acquisition procedure is usually met with a collection of degrading factors such as optical diffraction, comparative motion, under-sampling, and system noise. Generally, we assume that the procedure of image acquisition consists of warping, blurring, down-sampling, and noise degradations as shown in Fig. 1, and the observation model is definitely simulated the following:

$$\text{Eq.} \quad " y_k = DB_k M_k x + n_k " \quad (1)$$

where  $k$  is LR images that participated in the reconstruction process and  $x$  is the original image that degraded by warping ( $M$ ), blurring ( $B$ ), down-sampling ( $D$ ), and additive noise ( $n$ ). After the model is well known, an inverse process may be used to recover an HR image from a various of LR images. It can be an inverse problem that requires prior information from the HR image to get the reliable solution (Park et al., 2003; Protter et al., 2009).

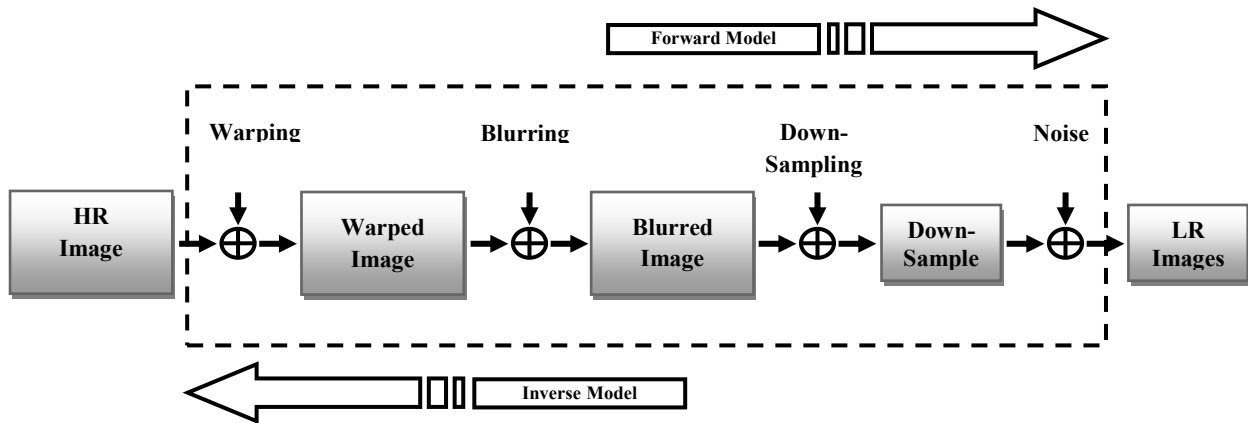


Fig. 1. The observation model employed in most SR techniques

### 3. Multi-images Super-resolution Approaches

As we discussed earlier, The essential intention of SR image reconstruction is certainly to generate a powerful HR image dependent on a few LR images that are captured through the exact same scene. There are numerous of various approaches regarding rebuilding the brand new HR image through the discovered LR images. These kinds of approaches attempt to address the particular aliasing artifacts that are generally contained in the LR images due to the under-sampling process by emulating the particular image observation model. In this paper, the SR image approaches can be categorized into three classes: (i) frequency-domain approaches, (ii) interpolation-based approaches, and (iii) regularization-based approaches as shown in Fig. 2. These varieties of approaches are certainly studied within the following subsections.

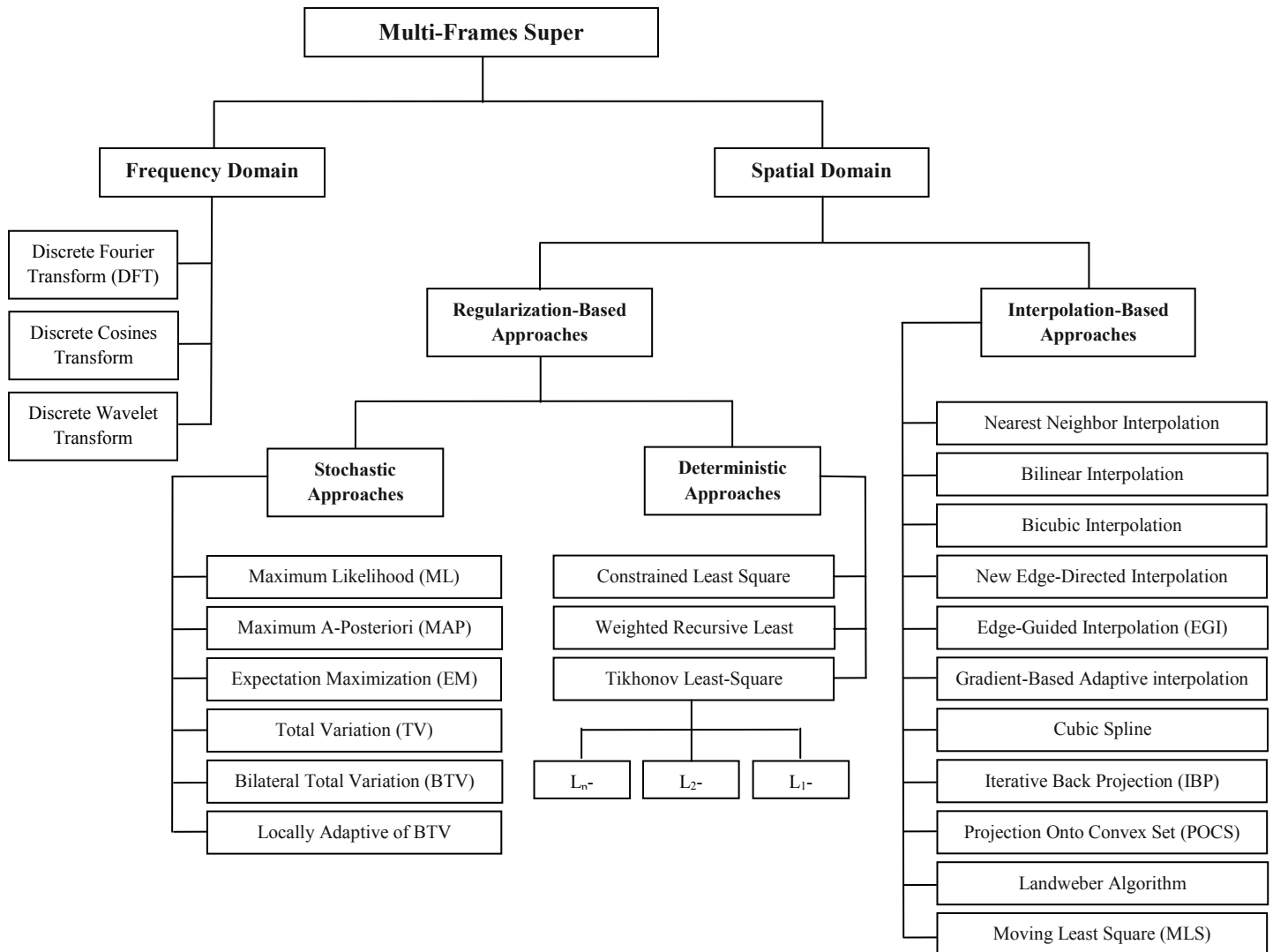


Fig. 2. The most popular multi-frames SR approaches

### 3.1. Frequency Domain Approaches

The frequency domain approaches are really a well-known way for enhancing the resolution of an image. In support of solving the particular issue of SR image reconstruction, frequency domain approach can certainly make specific use associated with the aliasing artifacts which usually are present in every LR image in order to rebuild the desired HR image. This certainly is achieved through transforming the input LR images towards the frequency domain and after that estimating the reconstructed HR image in this particular domain. Lastly, the reconstructed HR image is transformed back to the spatial domain. Actually, the first SR approach is created by Tsai and Huang (Tsai & Huang, 1984) based on the frequency domain for focusing on LR satellite images. A lot of researchers have got subsequently extended this approach to produce different types of SR approaches. These approaches are generally divided into three categories: discrete Fourier transform (DFT), discrete cosines transform (DCT) and discrete wavelet transform (DWT), which are described in the following subsections.

### 3.1.1. Discrete Fourier Transform

Tsai and Huang (Tsai & Huang, 1984) supposed that the series of LR images are globally translated and totally free from distortions such as blurring or noise effects. First, they suggested transforming the LR images information into the DFT domain and mixing all of them based on the relationship amongst the aliased DFT parameters of the detected LR images and the unidentified HR image. Second, the mixed data are converted back again to the spatial domain where in fact the new image could have an increased resolution than that of the insight images. An expansion of approach (Tsai & Huang, 1984) for a blurring and noising image was presented by Kim et al. (S. Kim, Bose, & Valenzuela, 1990), where they presented a weighted recursive least square algorithm dependant on the aliasing relationship among the LR images and HR image. Within their approach, the assumption is that all LR images include the exact blurring and the exact noising characteristics. This technique was further enhanced by Kim and Su (S. P. Kim & Su, 1993) to check out distinct blurs for every LR image. Bose et al. (N. Bose, Kim, & Valenzuela, 1993) suggested the recursive total least squares approach for SR reconstruction to minimize the negative effects of registration errors.

Vandewalle et al. (Vandewalle, Süsstrunk, & Vetterli, 2006) typically made use of correlation method in the frequency domain to discover motion parameters. The motion parameters are approximated depending on the actual fact which usually spatially shifted images in the frequency domain vary only with a phase shift. This phase shift among the two images is actually acquired coming from their correlation. Utilizing the phase correlation method, each of the image rotation and scale is transformed into vertical and horizontal shifts. To reduce errors generated by aliasing, the small portions of the discrete Fourier parameters are used because of it are free from aliasing. After that, the LR images are mixed based on the relationship among the aliased DFT parameters of the noticed LR images and the unidentified HR image. After fusion, the information is converted back again into the spatial domain and obtained the reconstructed HR image (Vandewalle et al., 2006).

### 3.1.2. Discrete Cosines Transform

DCT method was developed by Kang and Rhee (Rhee & Kang, 1999). They decrease the memory requirements and computational costs by applying DCT rather than DFT. In addition, they apply the multi-channel adaptive regularization parameters to eliminate ill-posedness such as underdetermined instances or unsuitable motion information instances. Park et al. (Park, Kang, Segall, & Katsaggelos, 2004) suggested a HR reconstruction technique for DCT depending on compressed images that concurrently approximates the quantization system noise. In order to really simplify this, they make the quantization noise in the spatial domain as a colored Gaussian noise process, plus they obtain the inverse noise covariance matrix to create a multi-channel smoothing functional. The suggested inverse noise covariance matrix differs with the signal pattern, and it prevents a demand to make the original DCT coefficients at low bit-rates. Kumar (Kumar, 2011) proposed a DCT approach with the reliable denoising, which regularly rebuilds a HR image from a few of LR images.

### 3.1.3. Discrete Wavelet Transform

Recently, a large number of researchers started to research the utilization of the wavelet transformation for handling the SR problem to extract the complete details which usually is

dropped or degraded through the procedure of the image acquisition. This is exactly encouraged by that the wavelet transformation offers a strong and effective multi-scale representation of the image for retrieving the high-frequency details (Nguyen & Milanfar, 2000). This method commonly treats the noticed LR images as a low-pass filter subbands of the unidentified wavelet transformation HR image. The goal is to approximate the greater scale subband coefficients, accompanied by employing the inverse wavelet transformation to create the HR image.

Nguyen and Milanfar (Nguyen & Milanfar, 2000) used wavelet interpolation accompanied by restoration technique for SR. They initially computed the wavelet coefficients of LR images, after that interpolated them for blurred values at the HR grid points. Through deconvolving, the interpolated values with the known blurring, an estimation of HR image is achievable. El-Khamy and et al. (El-Khamy, Hadhoud, Dessouky, Salam, & El-Samie, 2005) executed the registration of multiple LR images in the wavelet domain. Wavelet coefficients are denoised and merged after registration utilizing a regularization method. Interpolation strategies are used to obtain HR wavelet coefficients. Lastly, an inverse wavelet transform was executed to obtain the HR image in the spatial domain. Chappalli and Bose (Chappalli & Bose, 2005) additionally applied very soft thresholding methods to eliminate the noise from the wavelet coefficients to build up a real-time denoising and SR reconstruction strategy. Ji and Fermüller (Ji & Fermüller, 2006, 2009) suggested a powerful wavelet SR method of tackle the mistake incurred in both the registration computation and the blurring detection computation. They break down the wavelet coefficients directly onto two channels. Finally, these coefficients seemed to be upsampled, filtered, and merged to obtain the simulated image. The SR image was gathered using iterative back projection technique with effective regularization conditions at each iteration to eliminate the noise. Li (Xin Li, 2007) suggested image resolution improvement by extrapolating high-band wavelet coefficients.

Anbarjafari and Demirel (Anbarjafari & Demirel, 2010) proposed an exciting new SR technique depending on interpolation of the high-frequency subband images acquired by DWT and the input LR image. The suggested technique takes advantage of DWT to decompose an image into several subband images. Then the high-frequency subband images and the input LR image are generally interpolated accompanied by merging each one of these images to obtain a new HR image by using inverse DWT.

Zadeh and Akbari (Zadeh & Akbari, 2012) offered a multi-wavelet and cycle-spinning based on improvement approach to increase the image resolution. The proposed approach produces a HR image for the input LR images using the input images and an inverse multi-wavelet transform. Panda and Jena (Panda & Jena, 2016) taken into account the wavelet transformation to regenerate the enhanced image. Additionally, the genetic algorithm can be used to smooth the noise and obtain an ideal SR image.

### **3.2. Spatial Domain Approaches**

Spatial domain approaches are classified as the most popular to develop the SR image. The popularity of these approaches is due to the motion is not restricted to translational shifts only and therefore a more general global or non-global motion may also be integrated and managed. In this paper, spatial domain approaches are usually split into interpolation-based

approaches and regularization-based approaches, which are explained in the following subsections.

### 3.2.1. Interpolation-Based Approaches

These approaches are the most intuitive techniques for constructing the SR image based on firstly the projection of all the obtained LR images into the reference image. After that, all the details obtainable from every image are merged, because of all LR images present quantity of extra details regarding the scene. Lastly, the image is deblurred for creating the SR image. The interpolation-based approaches consist of the following three steps: registration of LR images, interpolation into the HR grid, and restoration of HR image. First, the image registration step is the procedure of geometrically aligning a group of LR images of the exact scene with regards to one specific LR image named the reference image. LR images contain distinct sub-pixel shifting and rotations from each other. Therefore, it is necessary to obtain a correct approximation of movement parameters before merging them to generate a HR image. Incorrect estimation of movement parameters results in a variety of visual artifacts that as a result degrade the resolution of the rebuilt image. Second, the image interpolation step is used for generating a HR image by estimating new pixels in the image's given a group of pixels. Final, the image restoration step is used for improving the reconstructed HR image that is created in the interpolation step. The image interpolation has an essential role in estimating a HR image. There are various interpolation methods.

The most simple approach for image interpolation is the nearest neighbor interpolation (Nguyen, Milanfar, & Golub, 2001). For every pixel on the HR grid, the nearest known LR pixel is chosen and the value of this pixel is merely used as the value at the grid point. It's the fastest of most interpolation approaches as it considers only an individual pixel that nearest to the grid point being interpolated. However, this approach is able to bring the significant distortion, show up the mosaic, and produce images with a blocky visibility. A second simple and well-known approach is the bilinear interpolation. Bilinear interpolation (Xin Li & Orchard, 2001) considers the nearest 2x2 neighborhood of known pixel values around the unidentified pixel. After that, it requires a weighted average of the 4 pixels to reach its last interpolated value. This approach leads to much smoother images than the nearest neighbor interpolation approach. However, bilinear interpolation approach is more complicated than the nearest neighbor interpolation approach, therefore it has bigger computation. It does not has any gray discontinuity problems and has sufficient results. This approach has a low pass filtering characteristics, therefore the high-frequency component is passed and the image contour has some extent of fuzzy.

Bicubic interpolation approach is generally executed in the same manner of bilinear interpolation approach, through taking into consideration the nearest 4x4 neighborhood of known pixels. Since, they are at different distances from the unidentified pixel. Bicubic interpolation approach can obtain relatively clear image quality, however, it requires a greater amount of computation. Therefore, this approach generates visibly sharper images when compared to the previous two approaches. It perhaps gets the optimal mixture of processing time and output quality (Xin Li & Orchard, 2001). Additionally, this approach could be most widely used in a large number of image processing applications such as Adobe Photoshop, Adobe After Effects, Avid and Macromedia Final Cut Pro etc. New Edge Directed

Interpolation (NEDI) (Xin Li & Orchard, 2001) is another approach where the interpolated pixels are approximated from the local covariance parameters of the LR images depending on the geometric duality among the LR and HR covariance.

Edge Guided Interpolation (EGI) approach (L. Zhang & Wu, 2006) splits the neighbor of every pixel to make a couple observation subsets through the orthogonal directions and estimate the lacking pixel. This approach merged both of these estimated values into the powerful estimation by applying linear-minimum mean square error estimation. Gradient-based adaptive interpolation (Chu, Liu, Qiao, Wang, & Li, 2008) additionally taking into consideration the distance among the interpolated pixel and the nearby respected pixel. The results illustrate that the suggested technique not only significantly increases and enhances the quality of recovered images, but also it is a powerful to detect the registration mistake and needs a low-computational cost.

Cubic spline approach (X. Zhang & Liu, 2010) meets a piecewise continuing curve and moving through lots of points. This spline contains weights that could be the parameters on the cubic polynomials. The fundamental job of the cubic spline interpolation approach is to compute weights that are used to interpolate the information. The registration, interpolation, and restoration steps in the SR approach can be executed to accomplish the HR image that comes from a series of LR images through the Iterative Back Projection (IBP) approach (Irani & Peleg, 1991). In IBP approach, the HR image is approximated by reducing the error among the simulated and observed LR images. This approach is extremely easy to understand and very simple. However, it is not generally going to give an unique result because of the ill-posed trouble. An additional simply implemented SR approach is the Projection Onto Convex Set (POCS) approach that developed by Stark and Oskoui (Stark & Oskoui, 1989). In POCS approach, a set of restrictions is described to limit the space of HR image. The restriction sets are curved and facilitate the particular attractive SR image features such as positivity, smoothness, bounded energy, and dependability. The intersection coming from all these sets represents the area of the allowable solution. As a result, this problem is minimized to locating the intersection of the restriction sets. the projecting operators are decided for every convex restriction set to get the solution. This operator reflects the primary estimation of the HR image against the relevant restriction set. Repetitively executing this method, a great solution is acquired at the area of intersection of the k convex restriction sets. This approach actually didn't integrate any observation noise.

In order to enhance the quality of an image, various methods are suggested to improved interpolation based approaches such as:

Ur and Gross (Ur & Gross, 1992) executed a non-uniform interpolation of a couple of spatially shifted LR images through the use of the generalized multi-channel sampling theorem. The benefit of this method is the low-computational cost, which is actually ideal for real-time applications. However, the ideality of the whole rebuilding process is not assured, because the interpolation mistakes are not considered. Komatsu et al. (Komatsu, Aizawa, Igarashi, & Saito, 1993) shown a scheme to obtain a better resolution image through the Landweber algorithm from multiple images that used concurrently with multiple cameras. In addition, they make use of the block-matching approach to measure comparative shifts. If the cameras currently have the same aperture, however, it enforces serious restrictions both in their agreement and in the configuration of the scene. Bose and Ahuja (N. K. Bose & Ahuja,



2006) made use of the moving least square (MLS) approach to approximate the intensity value at each pixel position of the HR image through a polynomial estimation using the pixels in a precise neighborhood of the pixel position in mind.

### 3.2.2. Regularization-Based Approaches

Generally, the SR image reconstruction approaches are really an ill-posed problem due to an inadequate number of LR images and also the ill-conditioned blur operators. Techniques which usually used to support the inversion of the ill-posed problem are identified as regularization. The regularization approach takes advantage of the prior knowledge of the unidentified HR image to resolve the SR problem. Deterministic and stochastic regularization approaches are offered in the following subsection.

#### 3.2.2.1. Deterministic Approaches

The deterministic approach presents the regularization term which transforms the ill-posed problem into a well-posed one. It happens through the use of prior information about the perfect solution depending on the regularization term ( $R$ ) and regularization constant ( $\lambda$ ) (Plenge et al., 2012). The constrained least square (CLS) regularization approach incorporates the smoothness constraints as a priori information. In this instance,  $R$  is the high pass filter which usually reduces the amount of high-frequency details in the new reconstructed image. The regularization parameter  $\lambda$  handles and controls the high-frequency information. The larger values of  $\lambda$  may possibly smooth the new reconstructed image which is a suitable choice only if a little number of LR images are present and/or there's a great deal of noise. The smaller values of  $\lambda$  might produce a noisy solution which can be applied when a huge quantity of LR images are present and the quantity of noise is small (Park et al., 2003). Tikhonov least-square approach (Plenge et al., 2012) incorporates  $l_2$ -norm of the second order derivation ( $p=2$ ) of the HR image as a regularization term. The primary benefit of the  $l_2$ -norm is that it is certainly easy to resolve. On the other hand, The  $l_2$ -norm doesn't promise a unique solution and it is optimum when the model error is white-Gaussian distribution (Plenge et al., 2012; Song, Zhang, Wang, Zhang, & Li, 2010). For this reason, Farsiu et al. (Farsiu, Robinson, Elad, & Milanfar, 2004) employed an alternative  $l_1$ -norm ( $p=1$ ) and verified that the  $l_1$ -norm works more effectively than the  $l_2$ -norm when the images consist of non-Gaussian errors for very quickly and powerful SR. Several researchers employed developed approaches (Shen, Peng, Yue, Yuan, & Zhang, 2016; Yue, Shen, Yuan, & Zhang, 2014; Zeng & Yang, 2013) with combined error modes. Therefore, the  $l_p$ -norm function ( $1 \leq p \leq 2$ ) may also be utilized as the constraint function due to its convex property and its own convenience for the imaging model errors [81]. When  $1 \leq p \leq 2$ , it leads to a weighted mean of measurements. If the value of  $p$  is near to one, then the solution is computed with a greater weight throughout the measurements near to the median value. When the value of  $p$  is near to two, the solution is estimated to the average value (Farsiu et al., 2004). Sometimes, images are infected by Gaussian and non-Gaussian errors, and the  $l_p$ -norm function is recognized to be a highly effective solution (Shen et al., 2016). Kim and Bose (S. Kim et al., 1990) suggested a weighted recursive least square algorithm for generating the SR image. The weight depends upon the prior information of the image. This algorithm provides higher weights to the LR images. With various weights, the problem basically decreases to the

general least square estimation. Finally, interpolation and restoration are adapted to get the HR image. Mallat and Yu (Mallat & Yu, 2010) suggested a regularization SR approach which uses adaptive estimators acquired by combining a family group of linear inverse estimators.

### 3.2.2.2. Stochastic Approaches

Depending on the observation model explained above, the goal is to rebuild the HR image from a collection of warped, blurred, noisy, and under-sampled images. As the model in (2) is an ill-conditioned, SR actually is an ill-posed inverse problem. As a result, the stochastic approach is well-known, especially the Bayesian theorem, because it presents the adaptable, flexible, and convenient way to incorporate a priori information and create a powerful relationship between the LR images and the unidentified HR image (Belekos, Galatsanos, & Katsaggelos, 2010; Pickup, Capel, Roberts, & Zisserman, 2009; Polatkan, Zhou, Carin, Blei, & Daubechies, 2015; Tian & Ma, 2010; Villena, Vega, Babacan, Molina, & Katsaggelos, 2013; Villena, Vega, Molina, & Katsaggelos, 2009; Woods, Galatsanos, & Katsaggelos, 2006).

Maximum likelihood (ML) is suggested by Tom and Katsaggelos (Tom & Katsaggelos, 1995). The goal of this approach is to get the ML estimation of the HR image. on the other hand, it only takes into account the relationship amongst the LR images and primary HR image. Therefore, Maximum A-Posteriori (MAP) method combines the prior image model to expose the expectancy of the unidentified HR image. ML and MAP approaches are famous because of their flexibility and adaptability with edge protecting and combined parameter approximation (Belekos et al., 2010; Pickup et al., 2009; Polatkan et al., 2015; Tian & Ma, 2010; Villena et al., 2013; Villena et al., 2009; Woods et al., 2006). Tipping and Bishop (Tipping & Bishop, 2002) employed the expectation-maximization (EM) algorithm to approximate the hyper-parameter value by increasing its misfit likelihood function. However, the EM algorithm produces a huge computational load, and moreover, it doesn't always meet to the global ideal (Wu, 1983). Pickup et al. (Pickup, Capel, Roberts, & Zisserman, 2006) developed the technique (Tipping & Bishop, 2002) by minimizing its computational cost and additionally altered the prior image model by taking illumination adjustments among the collected LR images. He and Kondi (He & Kondi, 2004) suggested a strategy to simultaneously estimation both hyper-parameter value and the HR image by enhancing the cost function. On the other hand, this approach considers that shifts between LR images are limited to integer value shifts on the new HR grid. In (Woods et al., 2006), Woods et al. suggested to use EM algorithm and calculate ML approximation of the hyper-parameter.

Total variation (TV) regularization is initially suggested by Osher et al. (Goldstein & Osher, 2009; Osher, Burger, Goldfarb, Xu, & Yin, 2005; Pan & Reeves, 2006; Yuan, Zhang, & Shen, 2012) to protect edge information and prevent ringing results. On the other hand, the finding results of the TV prior model is sometimes lead to a “staircase” result with intense noises, particularly in flat or smooth areas (Yuan, Zhang, & Shen, 2013). Several researchers are suggested spatially adaptive approaches for overcoming the disadvantages of the TV prior model. Several approaches apply spatially adaptive regularization parameters to remove the staircase results (Dong, Hintermüller, & Rincon-Camacho, 2011; Yuan et al., 2012). There are a few approaches that categorize the image into flat and detailed areas by spatial

information and also use a greater and a smaller charges parameter for the smooth regions and edges respectively. On the other hand, the spatially adaptive indications such as the difference curvature, gradients, and structure tensor are usually very critical to the noises. Bilateral total variation (BTV) (Farsiu et al., 2004) that is utilized for estimating TV, maintains the flatness of continued areas, and protects edges in discontinued areas. a locally adaptive version of BTV (LABTV) is presented for giving a stability among the noise reductions and the protection of image information (Xuelong Li, Hu, Gao, Tao, & Ning, 2010).

#### 4. Similarity Measure

To be able to assess the fidelity of the image reconstruction procedure, every reconstructed HR image has to matched to the original image which is called similarity measurement. In addition, it assists in the monitoring and evaluating the performance of the image reconstruction procedure. There are numerous similarity measures existing in literature. However, they are not agreed globally when matching two images. A few of the most well-known are Normalized Cross-Correlation Ratio (NCCR), Direct Difference Error (DDE), Peak Signal-to-Noise Ratio (PSNR), Root Mean Square Error (RMSE) and Structural Similarity (SSIM) (Begin & Ferrie, 2006; Tang, Deng, Xiao, & Yu, 2011; Wang, Bovik, Sheikh, & Simoncelli, 2004).

The PSNR is measured from the MSE, which is the average error amongst the original image and the SR image. Given a SR  $m \times n$  image  $\hat{X}(i, j)$  and its original  $X(i, j)$ , MSE and PSNR are defined as:

$$"MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [X(i, j) - \hat{X}(i, j)]^2 "$$
 Eq. (2)

$$"PSNR = 20 \log_{10} \left( \frac{L}{\sqrt{MSE}} \right) "$$
 Eq. (3)

The SSIM index measures the similarity between the SR and original images. The SSIM considers luminance, contrast, and structural adjustments amongst the two images. The SSIM index is defined as:

$$"SSIM(X, \hat{X}) = \frac{(2\mu_X \mu_{\hat{X}} + C_1)(2\sigma_{X\hat{X}} + C_2)}{(\mu_X^2 + \mu_{\hat{X}}^2 + C_1)(\sigma_X^2 + \sigma_{\hat{X}}^2 + C_2)} "$$
 Eq. (4)

where  $\mu_X$  and  $\mu_{\hat{X}}$  are the means and  $\sigma_X$  and  $\sigma_{\hat{X}}$  are the standard deviations of the original and SR images,  $\sigma_{X\hat{X}}$  is the covariance of  $X$  and  $\hat{X}$ , and  $C_1$  and  $C_2$  are constants. The value of SSIM closes to 1, if the SR image is very similar to its original image.

#### 5. Conclusion

In this paper, a general survey of the present multi-frame SR approaches are presented over the previous three decades. The primary improvement in SR approaches can essentially be split into three phases. In the first 10 years, researchers altered their concentration from the

study of frequency domain approaches to spatial domain approaches, especially interpolation-based approaches. Regularized SR approaches are the primary emphasis in next phase. The Bayesian MAP construction has become the most common approach because of its great performance and flexible properties within the last phase. Recently, researchers have primarily concentrated on SR reconstruction in the several areas. Nevertheless, the comprehensive practical use of SR still continues to be described as a big problem.

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