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Delaunay Triangulation of a missing points

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ABSTRACT

Surface reconstruction of scattered data points is one of the challenging area where the main purpose is to produce a smooth surface. In this research, Delaunay triangulation method was used to construct surface of scattered data points for six different test functions. In certain cases some surface producing holes after scanning where it becomes difficulty to produce a smooth surface. This research intends to test the accuracy of Delaunay triangulation in generating different surface when the points of scattered data were removed. The points removed were according to the percentage of points and the new surface was generated for every removing point. The result of the study shows interpolating surface produced by the removing points and the total absolute error with mean absolute error was calculated and compared.

Key words: **delaunay triangulation, surface, scattered, holes, absolute error.**

1. Introduction

Surface reconstruction is the automated process for generation of three dimensional surfaces from a point cloud of data of a real object acquired from a 3D scanning. Surface reconstruction was important in the area of industrial manufacturing, computer animation and medical visualization. In industrial manufacturing; engineers can inspect the quality of their manufactured parts by scanning them and comparing with the original 3D CAD model. On the other hand, animators can acquire real world surface and produce realistic scenes and environments. Doctors and scientists can use the data to design and manufacture medical devices and prosthetics. For example dentists can automatically manufacture teeth to replace cracked or damaged ones by scanning and reconstructing the surface of original tooth.

There are three main challenges for surface reconstruction which are noisy data; misalignment involves multiple scan, non-linear distortions and lack of featuring lead to imperfect alignment of scan. Third challenge is the missing data where some surface such as dark-coloured surface, glossy or hollow surface cannot be capture during scanning which leaves holes in the resulting data set (Bolitho, 2010; Lim and Haron,2014).Many methods can be used for scattered data fitting technique such as Radial Basis Function (RBF), Moving Least Square (MLS) and Delaunay triangulation method.

Radial Basis Function (RBF) which is the linear combination of radially symmetric basis function was used to define points that are not restricted to a regular grid and there is no need to define a mesh of patches (Saaban et. al, 2012). RBF have been applied in reconstructing incomplete surface that contain holes however it only apply to smooth surface, limited to the size of the holes and identification of holes need user interaction (Branch et. al, 2006). Disadvantage of RBF method due to the ill-conditioned when processing large data set, therefore single and multilevel quasi interpolation using Compactly Supported RBF (CS-RBF) was introduced (Liu and Wang, 2012). But CS-RBF still cannot handle point set that have sharp features.

Moving Least square (MLS) method which also a meshless method allows the fit to change locally and the solution changes with the value of x. MLS method have the ability to handle hole-filling problem, for example a research in (Shen, 2006) shows the application of moving least squares to a variety of three dimensional polygonal models which contains holes, self-intersections, non-manifold edges and other defects. Normally least square method do not need to go through all data points but MLS pass through all the data point so it shows that MLS provide better curve and surface fitting with interpolation conditions (Ni et. al,2010).

Delaunay triangulation is one of the popular methods used for generation surface of scattered data point. Delaunay triangulations were used to generate triangle meshes whose vertices are the sample points. Delaunay triangulation result in the triangulation that maximize the minimum angles of all triangles and circle that circumscribe three vertices of any triangle contain no other vertices (Grise and Meyer-Hermann,2011). It does not involve deletion of a sample points so it can preserve the topology of original surface. The objective of this research is to remove some points from Delaunay triangulation and re-triangulate again. The results of the study able to determine the mesh difference of Delaunay triangulation after remove some points.

2. Review of literature

Triangulated surface can be divided into two which are Delaunay triangulation and its dual Voronoi diagram. Delaunay triangulation were used to generate triangle meshes whose vertices are the sample points. The challenge for this technique is to determine the connectivity of the sample points so that it preserves the topology of original surface. There is also a need to produced a smooth surface of Delaunay triangulation.There are many techniques for constructing Delaunay triangulation of scattered data points such as incremental construction algorithm, sweep-line algorithm and circle-sweep algorithm.

Incremental Delaunay triangulations start with the addition of points one by one into the triangulation and ensure it satisfies empty circumcircle condition.(Freedman,2007) use incremental construction algorithm in a non-orientable surface such as Hilbert space, Klein bottle and real projective plane which cannot embedded in $\Re^3$ without self-intersection. The algorithm start by choosing a dangling edge from current simplices and add one simplex of surface at a time.

Sweep-line algorithm which a line move over the plane and stop at two event point, site event and circle event (Fortune,1987) . It uses the frontier to keep the list of half edge and depend on priority queue when the sweep line move and stop. The method was good but spends more time on searching event queue. Second method by (Zalik,2005) of sweep line involved using lower border and advancing front border and all vertices triangulated between the two borders must follow empty circle property. It use two heuristics to prevent tiny triangle where the first is legalized: walking left and right of advancing front and the other to remove basins. Basin means no point appears at advancing front so it takes longer times when this happen.

On the other hand, circle-sweep algorithm sweeping the plane with increasing circle of pole distance where the pole is the fixed points in the circle and reach two event which is site event and ultimate point event. (Biniaz,2012) correct the error of Zalik sweep-line algorithm by combining faster circle-sweep algorithm with Zalik sweep-line algorithm. The triangulated vertices surrounded by frontier and the shape of the frontier depend on arrangement of points already swept. It reduced legalization and reduces the number in-circle tests and edge flips. Limitation of the research is on the value of constant k, the hash table restores the process will be worst if k is close to one or too large.

Delaunay triangulation required the triangulation to satisfy convex quadrilateral properties where sum of opposite angle for four vertices must be less than or equal 180^0 in order to use the edge flipping technique (Jiang et al., 2010).This can be achieved by applying either three optimization criterion mentioned by (Lawson,1977) for convex quadrilateral which are max-min angle criterion which choose triangulation that maximize minimum interior angle of two adjacent triangle. Secondly is circle criterion where a circle surrounding three vertices of convex quadrilateral should check whether the fourth vertex is in interior or exterior of the circle then edge flip need to be done. Thirdly is Thiessen region criterion where the thiessen region surrounding each point should be strong thiessen region which connected to each other and not contact at one point only. The edge flipping technique have been proven by (Osherovich and Bruckstein ,2008) where the shared edge of two adjacent triangle can be replace with other diagonal if it was a convex quadrilateral. Mathematical induction was conducted in 2 cases where one of the flip closures was done into empty polygonal and the others with points.

One research by (Cheng and Jin, 2013) use the cocone algorithm to reconstruct a triangular mesh and then random edge flips was performed. Repeated edge flips can lower the circumradius of every mesh triangle, reduce the normal deviation and the dihedral angles in mesh closer to π. Another characteristic of Delaunay triangulation was mentioned in a research by (Menaka et al.,2010) which focus on comparison of surface reconstruction by Constrained Delaunay triangulation with the standard marching cube algorithm on the anatomical facial reconstruction from 2D CT slices. The research focused to ensure its satisfy Delaunay criterion which is no point in a set of points lies inside circumcircle of any triangle. Advantages of Delaunay Triangulation are triangulation can help to maximize the minimum angle of all angles of the triangle in triangulation and avoid skinny triangles. It does not involve deletion of a sample points so it can be used in variety of application and it can preserve the topology of original surface.

3. Methodology

The technique used in this research was to remove some points from the original control points and generate a surface by using six test functions. The mesh difference was collected for 5 difference candidate. Fig. 1 below shows the flow chart for the research.

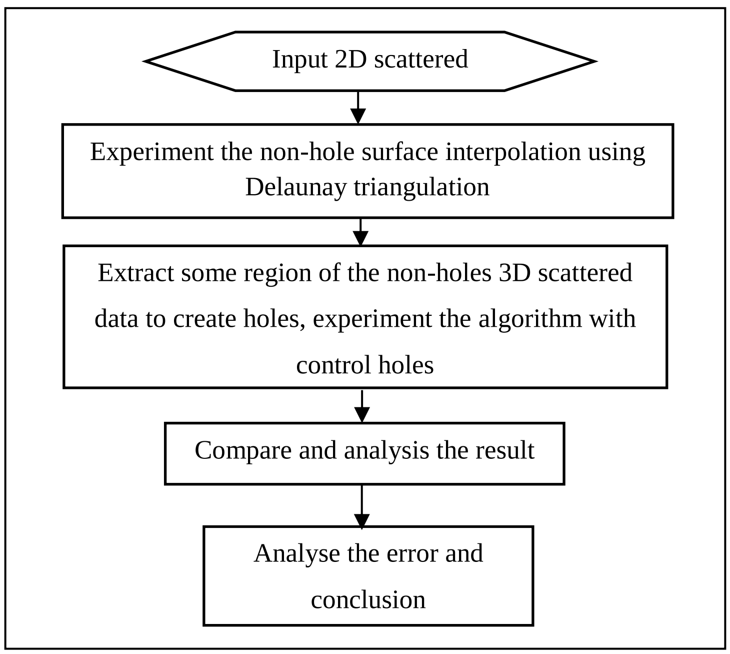

Fig.1. Flow chart for the research

The test domain of 36 data points used was by (Whelan,1986). These points are triangulated using Delaunay triangulation method. TABLE I shows the coordinates of these 36 points.

TABLE 1: 36 DATA POINTS

No	Coordinates		No	Coordinates	
	x	y		x	y
1	0.00	0.00	**19**	0.80	0.85
2	0.50	0.00	**20**	0.85	0.65
3	1.00	0.00	**21**	1.00	0.50
4	0.15	0.15	**22**	1.00	1.00
5	0.70	0.15	**23**	0.50	1.00
6	0.50	0.20	**24**	0.10	0.85
7	0.25	0.30	**25**	0.00	1.00
8	0.40	0.30	**26**	0.25	0.00
9	0.75	0.40	**27**	0.75	0.00
10	0.85	0.25	**28**	0.25	1.00
11	0.55	0.45	**29**	0.00	0.25
12	0.00	0.50	**30**	0.75	1.00
13	0.20	0.45	**31**	0.00	0.75
14	0.45	0.55	**32**	1.00	0.25
15	0.60	0.65	**33**	1.00	0.75
16	0.25	0.70	**34**	0.19	0.19
17	0.40	0.80	**35**	0.32	0.75
18	0.65	0.75	**36**	0.79	0.46

The Delaunay triangulation mentioned above using six well-known test functions according to (Renka and Cline,1984) as in Eq.1- Eq.6 below.

1. Franke's exponential function

$$F_1(x,y) = 0.75e^{-\left(\frac{(9x-2)^2+(9y-2)^2}{4}\right)} + 0.75e^{-\left(\frac{(9x+1)^2}{49}+\frac{9y+1}{10}\right)} + 0.50e^{-\left(\frac{(9x-7)^2+(9y-3)^2}{4}\right)} - 0.20e^{-\left((9x-4)^2+(9y-7)^2\right)} \quad (1)$$

2. Cliff function

$$F_2(x,y) = \frac{\tanh(9y-9x)+1}{9} \quad (2)$$

3. Saddle function

$$F_3(x,y) = \frac{1.25+\cos(4.5y)}{6+6(3x-1)^2} \quad (3)$$

4. Gentle function

$$F_4(x,y) = \exp\left(-\left(\frac{81}{16}\right)\left((x-0.5)^2+(y-0.5)^2\right)\right)/3 \quad (4)$$

5. Steep function

$$F_5(x,y) = \exp\left(-\left(\frac{81}{4}\right)\left((x-0.5)^2+(y-0.5)^2\right)\right)/3 \quad (5)$$

6. Sphere function

$$F_6(x,y) = \sqrt{64-81\left((x-0.5)^2+(y-0.5)^2\right)/9-0.5} \quad (6)$$

4. Results and Discussion

Fig. 2 represent the result of 2D Delaunay triangulation with removing of 15%(5 points), 30%(11 points), 45%(16 points), 60%(22 points) and 75%(27 points). As seen in Fig. 2, Delaunay triangulations triangulate the removing vertices and represent a different triangular mesh for 2D data points. Fig. 3 through Fig. 8 are the mesh generated by original and removing points for different test functions. Fig. 3 show the difference of mesh surface of original points and removing points for exponential function (refer Eq. 1) where the surface was decrease proportional to the x value and some parts of the surface was missing due to the removing points.Fig. 4 shows the difference of mesh surface of original points and removing points for cliff function (refer Eq. 2). The surface mesh becoming flat when there is more points removed such as the difference between the original control points and the 16 remove points.Fig. 5 shows the difference of mesh surface of original points and removing points for saddle function (refer Eq. 3). The maximum saddle point was decreasing respectively to the points removed.

Fig. 6 shows the difference of mesh surface of original points and removing points for gentle function (refer Eq. 4). Fig. 7 shows the difference of mesh surface of original points and removing points for steep function (refer Eq. 5). Gentle and steep function shows the same rate of decrease when some points are removed, the height of the surface was decrease and some surface on the side of surface missing.Fig. 8 shows the difference of mesh surface of original points and removing points for sphere function (refer Eq. 6). The result of the z value for sphere function yield a complex double value for removing 5 points so the total absolute error and mean absolute error gives 0 values as in TABLE 2 and TABLE 3.

TABLE 2 shows the total absolute error for the original and removing points for 36 data points. The value of total absolute error was decrease when points remove in exponential function. On the other hand, the total absolute error was increase when points remove applied for other test functions. TABLE 3 shows the mean absolute error for the original and removing points for 36 data points. The mean absolute error was decrease for the exponential function while it increase for other test functions.

In the TABLE 2 and Table 3, the label of A, B, C, D and E was represented as:
A = difference between removing 15% and original points.
B = difference between removing 30% and original points.
C = difference between removing 45% and original points.
D = difference between removing 60% and original points.
E = difference between removing 75% and original points.

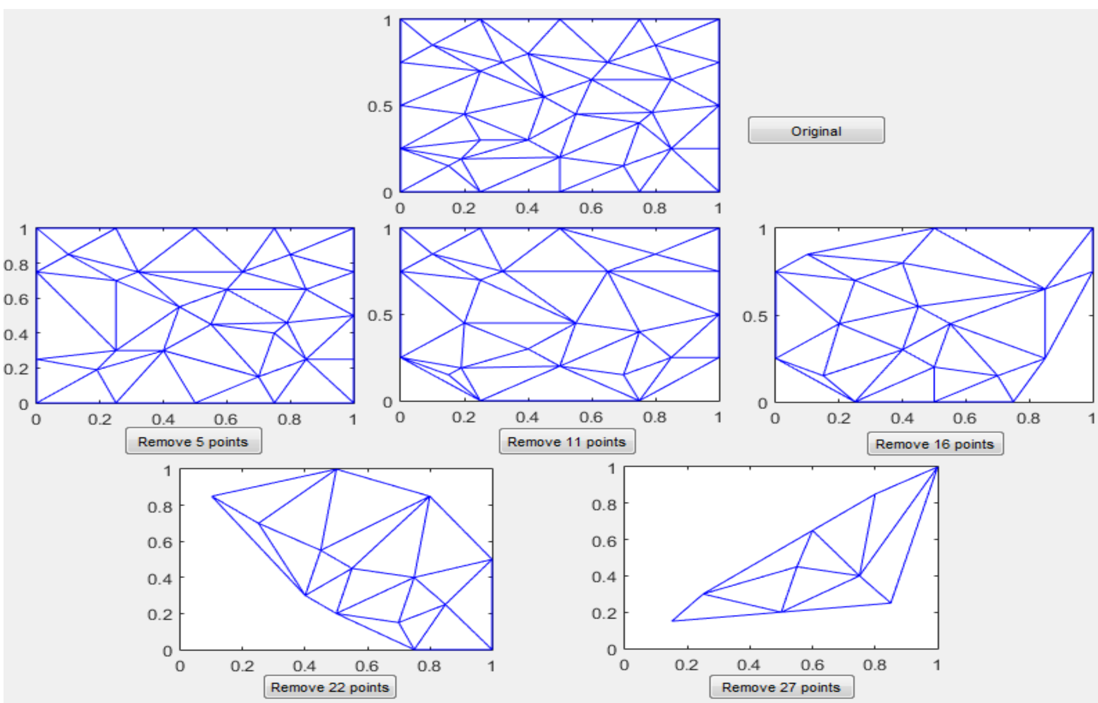

Fig. 2. 2D Delaunay Triangulation of original points and removing points.

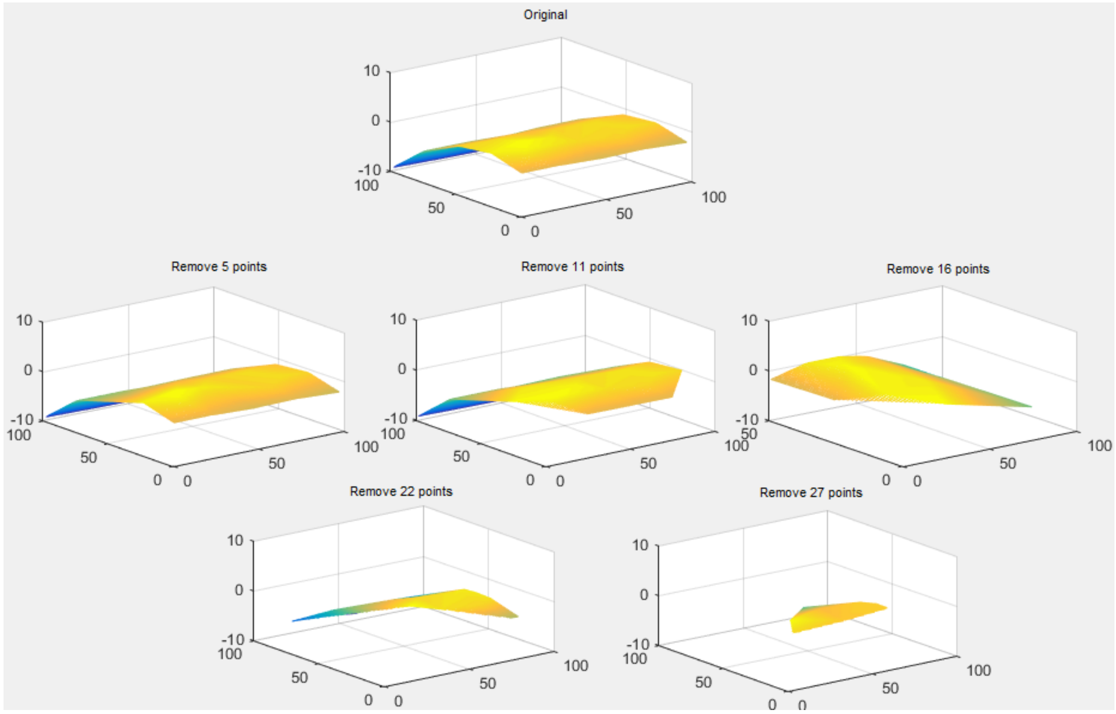

Fig. 3.Difference of mesh surface for exponential function

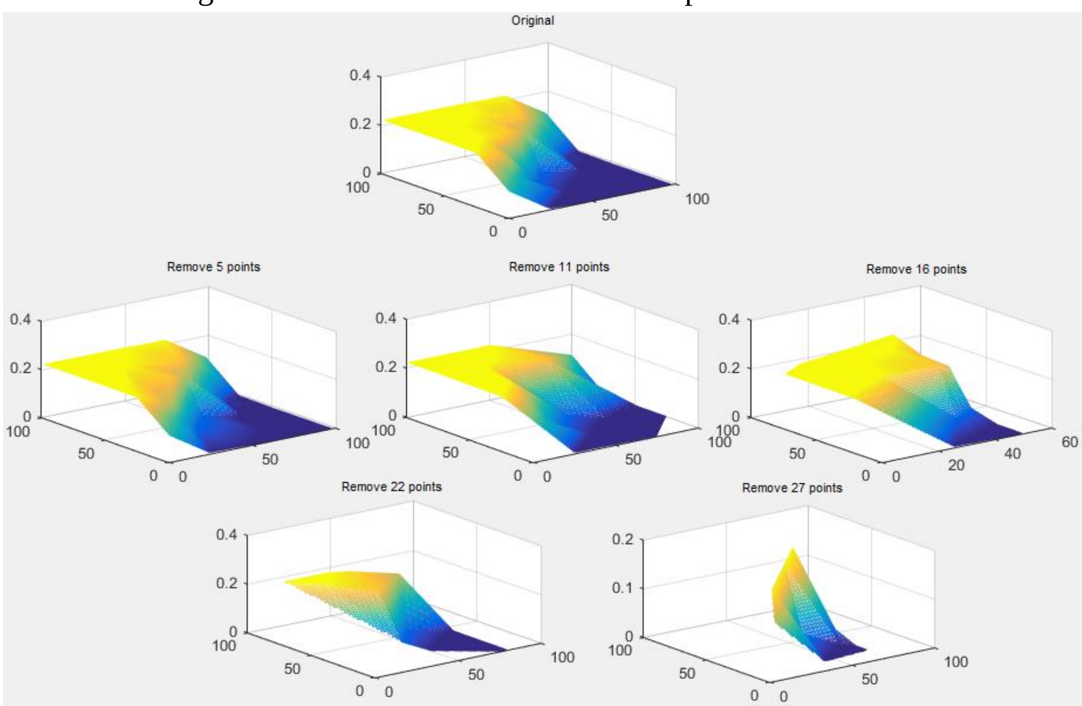

Fig. 4. Difference of mesh surface for cliff function

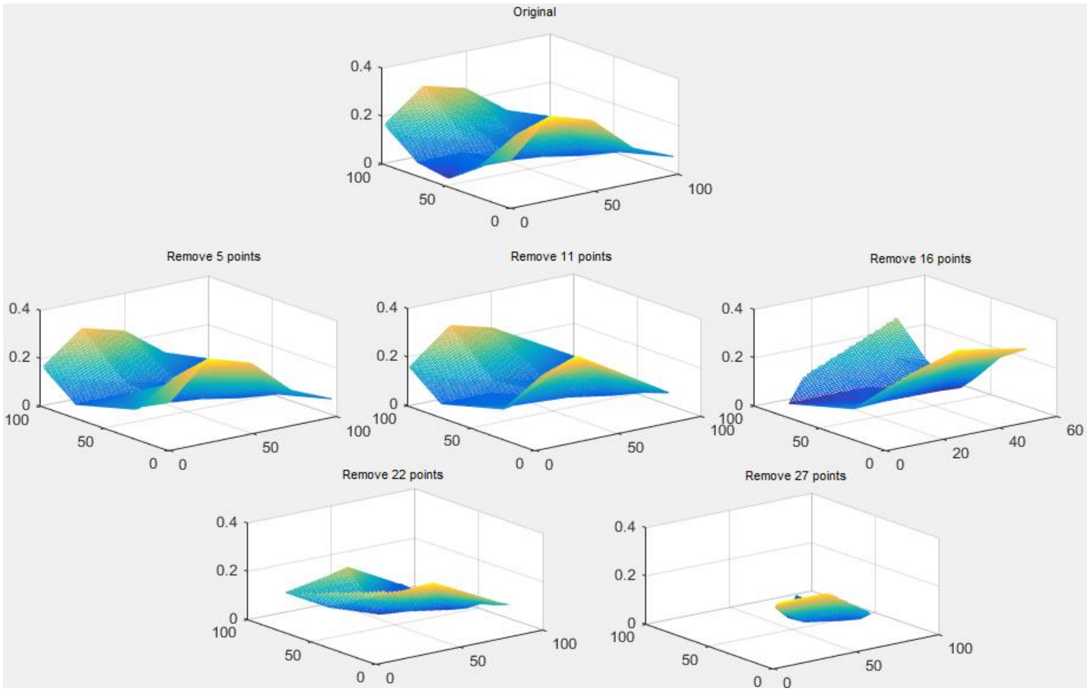

Fig. 5.Difference of mesh surface for saddle function

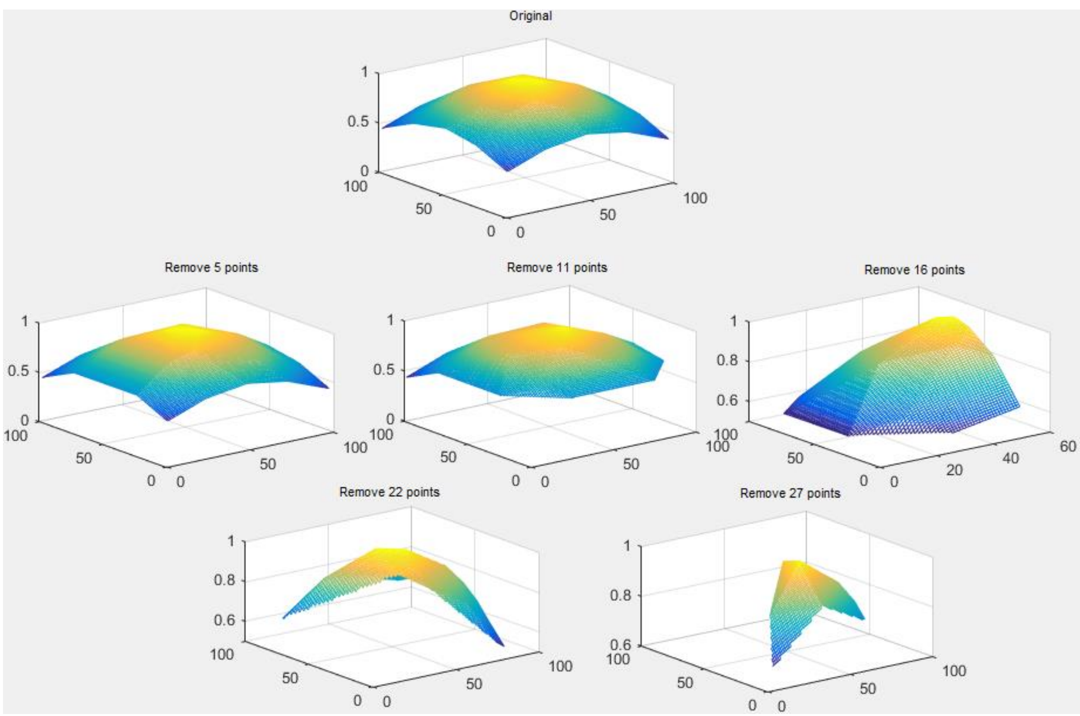

Fig. 6.Difference of mesh surface for gentle function

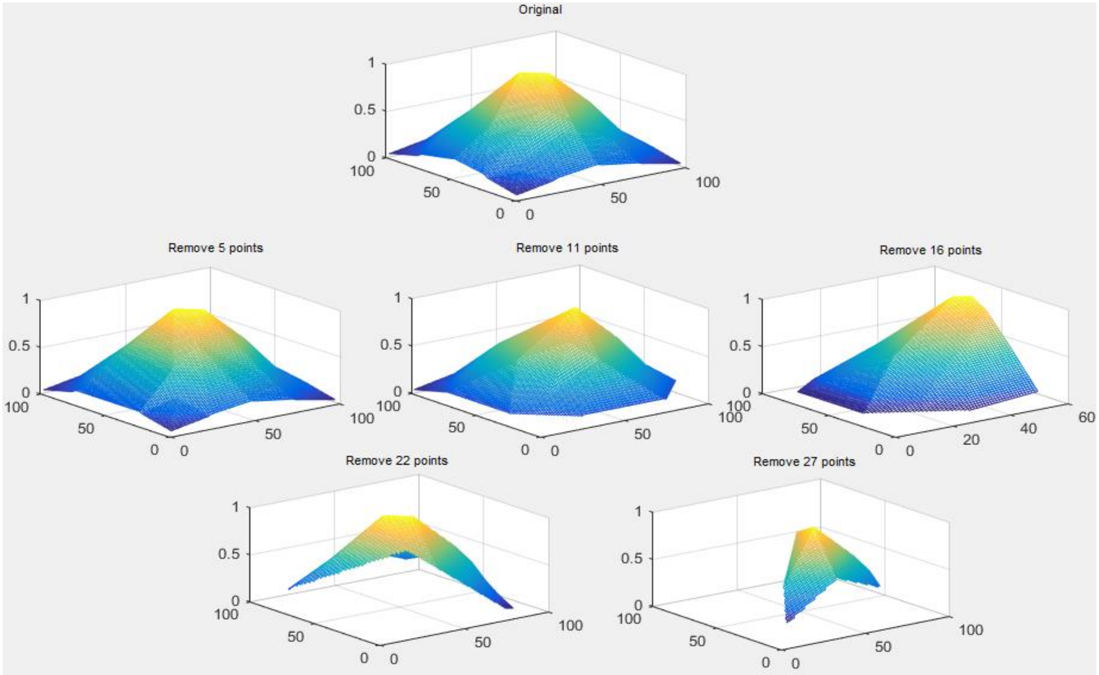

Fig. 7.Difference of mesh surface for steep function

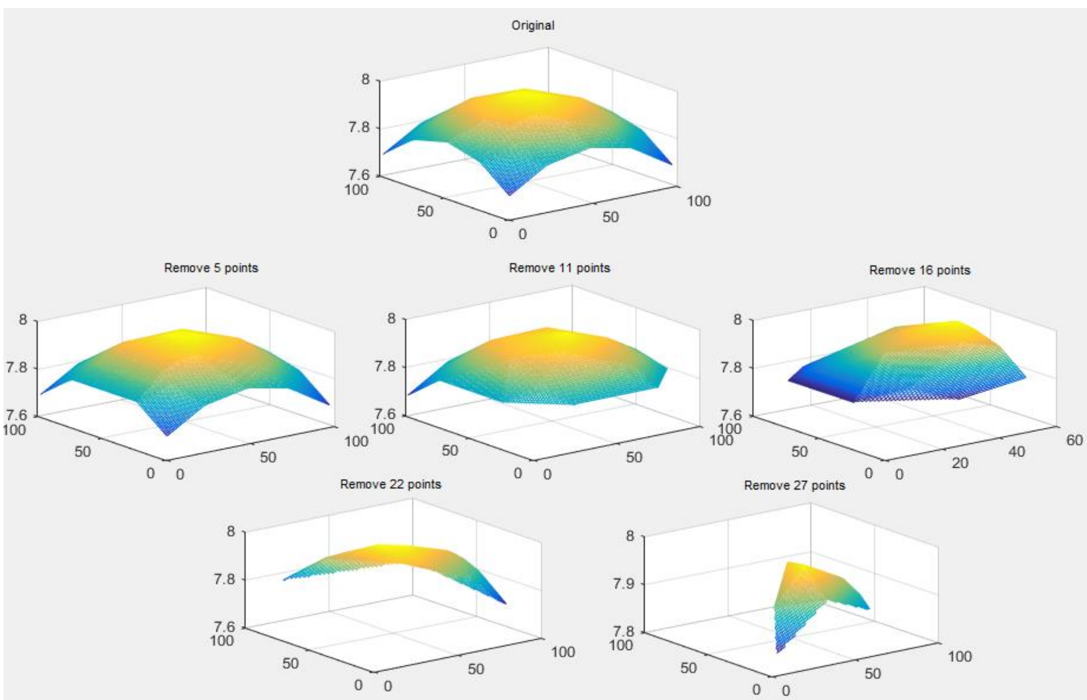

Fig. 8.Difference of mesh surface for sphere function

TABLE 2.TOTAL ABSOLUTE ERROR FOR EVERY TEST FUNCTION

Difference/ Functions	***Total Absolute Error***					
	F1	***F2***	***F3***	***F4***	***F5***	***F6***
A	7752.31	0.39	1.43	4.37	1.65	0
B	1881.36	0.68	1.17	5.73	3.56	54.41
C	1711.97	1.43	1.92	9.07	4.89	32.93
D	1217.92	1.84	2.28	10.82	6.22	109.64
E	306.96	2.74	3.95	15.09	6.70	117.14

TABLE 3.MEAN ABSOLUTE ERROR FOR EVERY TEST FUNCTION

Difference/ Functions	***Mean Absolute Error***					
	F1	***F2***	***F3***	***F4***	***F5***	***F6***
A	250.18	0.01	0.05	0.14	0.05	0
B	75.18	0.02	0.04	0.23	0.14	2.17
C	85.47	0.07	0.10	0.46	0.24	1.65
D	51.01	0.13	0.16	0.78	0.45	7.83
E	33.41	0.31	0.44	1.68	0.74	13.02

5. Conclusions

The result conclude that by removing more points, the surface of removing points are not consistent in shape and therefore this result conclude that for object with holes, the area should be introduced with more points representing the surface in order to produce a smooth surface and less.

6. Acknowledgment

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