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Prediction of residual stresses in cold formed corners

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Abstract:
This paper presents the investigation of residual stresses on the corners of cold formed high strength structural steel members. In order to achieve magnitude of residual stresses at inside corner of cold bent section, two-dimensional nonlinear finite element analyses (FEA) have been performed by simulating the press-braking process in Ansys software. These simulations were performed by defining material properties of two types of structural steels i.e. S355 and S650 while plate thickness of 5 mm and 10 mm has been used. During the analyses, r_o/t ratio of 4, 5, 6 and 7 for 5 mm plate thickness and r_o/t ratio of 6, 7 and 8 for 10 mm plate thickness with bent angle of 110^0, 100^0, 90^0, 80^0 and 70^0 have been maintained. As a result of theses analyses, the tension residual stresses obtained on the middle of the inside surface of the bend range from 60% to 92% of the yield strength of the material. These FE-simulations do not only provide the magnitude of residual stresses but also give a zig-zag type residual stress distribution pattern through the plate thickness. These results were then compared to the values predicted by equations proposed by Dat in 1980 which were found in good agreement.

Keywords: Residual stress, FEA, outer radius to thickness ratio r_o/t, bent angle

1. Introduction

The stresses in structural steels in their unloaded state are known as residual stresses (cruise et al., 2008). Residual stress in cold formed corners plays a significant role in determining their behavior and strength. Laboratory measurements of residual stress by destructive methods are not only time- consuming but also of limited tendency (Quach et al., 2006). Furthermore, high level of residual stresses in cold formed corners may be either beneficial of detrimental with respect to fatigue strength of the structure (Karren et al., 1967; Rondal et al., 1987; Kato et al., 1978; Webster et al., 2001). Hence, this paper explores the alternative approach of numerical prediction of residual stress in cold formed sections by performing FEM simulations of simple plate bending process with different r_o/t ratios and bent angles. Plate bending is a process of permanently deforming initially flat plates into desired configurations by the use of rolls or dies. The cold bending of steel plate involves plastic loading of the metal followed by an elastic unloading (Quach et al., 2006). It results in locking of residual stresses into the plate and the phenomenon of elastic recovery is termed as spring back as shown in figure (1). The inelastic loading stress, the elastic unloading stress, and the final state of residual stress for a typical cold bent steel plate is shown in figure (2).

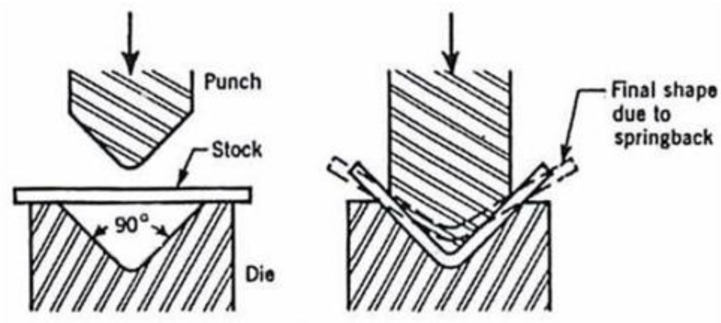


Figure 1 Plate Bending operation and springback due to unloading

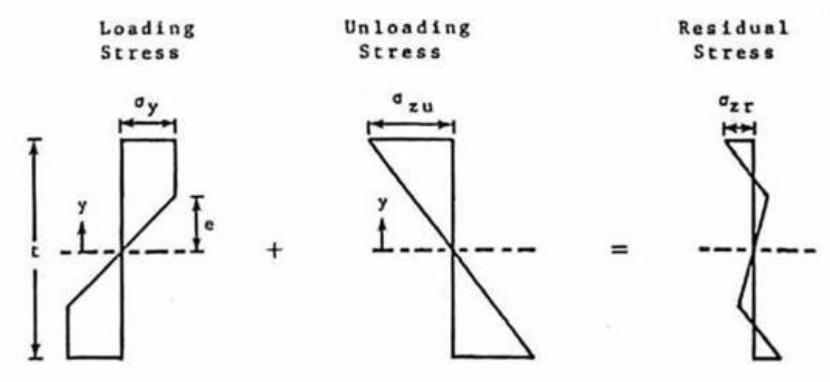


Figure 2 Loading, unloading and residual stress in cold bent section

2. Literature Review:

Ingvarsso investigated residual stresses in cold-bent thin plates considering the presence of three kinds of external loadings; end moments, tension forces and internal pressure (Ingvarsson et al., 1977). Dat (1980) predicted the residual stresses caused by severe cold-bending by developing equations. These equations were based on the results obtained from (Timoshenko and Goodier, 1970), which gives the loading and unloading stresses in the bending of a plate respectively. Residual stress is the sum of the loading and unloading stress. Few assumptions were made in the solutions which are as follows:

- the material is elastic-perfect plastic
- bending occurs under plane strain condition
- small plastic region near the neutral axis can be negligible

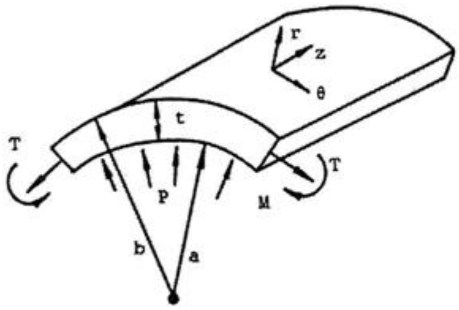


Fig.3 Bent Plate coordinates system

The coordinate system used by Dat (1980) shown in Figure 3 and the calculations for the residual stresses are as follows;

2.1 Loading Stresses

For $a \le r \le c$

$$\sigma_r = 2K\left(-p - \ln\frac{r}{a}\right) \quad (1)$$

$$\sigma_\theta = 2K\left(-p - 1 - \ln\frac{r}{a}\right) \quad (2)$$

For $c < r \le b$

$$\sigma_r = 2K\ln\frac{r}{b} \quad (3)$$

$$\sigma_\theta = 2K\left(1 + \ln\frac{r}{b}\right) \quad (4)$$

2.2 Elastic Unloading Stresses

Bending Unloading Stresses

$$\sigma_{rb} = 2K\left(-\frac{4M}{a^2N}\right)\left(\frac{b^2}{r^2}\ln\frac{b}{a} + \frac{b^2}{r^2}\ln\frac{r}{b} + \ln\frac{a}{r}\right) \quad (5)$$

$$\sigma_{\theta b} = 2K\left(-\frac{4M}{a^2N}\right)\left(-\frac{b^2}{r^2}\ln\frac{b}{a} + \frac{b^2}{r^2}\ln\frac{r}{b} + \ln\frac{a}{r} + \frac{b^2}{a^2} - 1\right) \quad (6)$$

Pressure Unloading Stresses

$$\sigma_{rp} = 2K\left(\frac{-a^2 p}{b^2 - a^2}\right)\left(1 - \frac{b^2}{r^2}\right) \tag{7}$$

$$\sigma_{\theta p} = 2K\left(\frac{-a^2 p}{b^2 - a^2}\right)\left(1 + \frac{b^2}{r^2}\right) \tag{8}$$

2.3 Residual Stresses

$$\sigma_{r'} = \sigma_r + \sigma_{rp} + \sigma_{rb} \tag{9}$$

$$\sigma_{\theta'} = \sigma_\theta + \sigma_{\theta p} + \sigma_{\theta b} \tag{10}$$

$$\sigma_{z'} = 0.5(\sigma_r + \sigma_\theta) + 0.3(\sigma_{rp} + \sigma_{rb} + \sigma_{\theta b} + \sigma_{\theta p}) \tag{11}$$

Where, 2K = σ_y for Tresca criterion, $2\sigma_y/\sqrt{3}$ for von Mises criterion; r = radius to an arbitrary point in the bend; a = radius to the inside of bend; b = radius to the outside of bend; c = radius to the neutral surface of the bend that can be found as c = $\sqrt{abe^{-p}}$; p = internal pressure applied over the inner surface of the bend: M = $\left(a^2 + b^2 - 2abe^{-p}\right)/4 - abp/2$; and N = $\left[\left(b^2/a^2\right) - 1\right]^2 . \left[\ln(b/a)\right]^2$.

Quach and Teng presented the results of research which forms part of a larger study on the theoretical predictions of residual stresses in cold formed sections (Quach et al., 2004). They investigated residual stresses by modeling the coiling and uncoiling process as an elastic plastic plane strain pure bending problem with steel assumed to obey the Von-Mises yield criterion and the Prandtl-Reuss flow rule. Rondal presented a similar numerical analysis of the pure plastic bending of wide plates and then proposed an approximate approach of deriving residual stresses in channel sections based on the results from his pure bending analysis (Rondal et al., 1987). Quach and Teng presented a finite element based method for predicting the residual stresses in press-braked thin walled sections (Quach et al., 2006). They analytically accounted the effects of coiling and uncoiling with the resulting residual stress described as the initial stresses in a subsequent finite element simulation of cold bending. Weng and Peköz measured longitudinal residual stresses in cold formed channel sections and found that the magnitudes of residual stresses in corner regions were higher than those in flat portions (web, flanges and lips) (Weng et al., 1990). They also observed that the inner surface of the section was subjected to tangential compressive residual stresses while the outer surface was subjected to tangential tensile residual stresses. Sami and Björk investigated the fatigue crack growth in residual stress field by experiments and numerical modeling of cold forming process from circular to rectangular hollow cross sections (Sami et al., 2004). The process modeling has been carried out as a continuous rigid-flexible non-linear contact analysis. They observed from fatigue experiments that both the fatigue crack growth path and the fatigue life are greatly influenced by the through thickness residual stresses. Weng and White presented experimental investigation of residual stresses in severely cold bent thick high strength steel plates (Weng et al., 1990). Their experimental work includes the measurement of residual stresses in cold-bent HY-80 and HY-100 steel plates of 18 (457.2 mm) in. square with 1 (25.4 mm) and 1.5 (38.1 mm) in. thickness. A total of 18 steel plates were tested with bend radii of 1.5 (38.1 mm), 2.5 (63.5 mm), 3.5 (88.9 mm), and 5.5 (139.7 mm) in., and bend angles of 90^0, 120^0, and 150^0. The yield strength of the material was 593 MPa (HY-80) and 721 MPa (HY-100). They found that tension residual stresses on the inside surface of the bend range from 46% to 92% of the yield stress of the material. They also observed zig-zag type residual stress distribution pattern through the plate thickness. These test results were compared to the values predicted by the equations proposed by Dat (Dat et al., 1980).

3. Modeling the plate bending process:

In order to obtain the residual stresses at inner corner of bend plate, two-dimensional non-linear finite element analyses have been performed in ANSYS software. This is done by modeling steel plates of 5 mm and 10 mm thicknesses by defining non-linear properties of S355 and S650 steel as shown in table 1.

Table 1 Material Data for Steel with T = 20°C and E = 210000 MPa

Steel Type	**Yield Strength, σ_y [MPa]**	**Elastic Strain, ε_e**	**Engineering Strain, ε_0**	**Ultimate Strength, σ_u [MPa]**	**True Stress, σ [MPa]**	**True Strain, ε**
S355	355	0.0016905	22%	630	768.60	0.19885
S650	650	0.0030952	16%	700	812	0.14842

In order to achieve accurate r_o/t ratio, proper die and punch along with other design parameters have been defined as shown in figure (4) and table 2 and table 3.

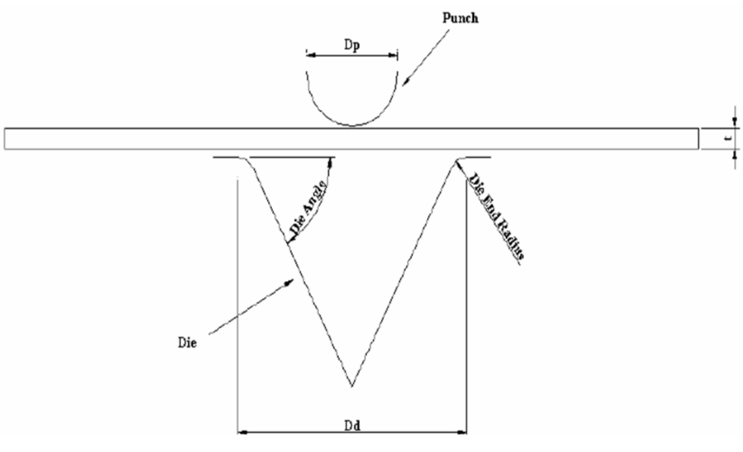


Figure 4 Air-bending process model geometry for ANSYS

Table 2 Design Parameters for Steel Plate with L = 200 mm and t = 5 mm

r_o/t	Outer Radius of the Tube, r_o [mm]	Inner Radius of the Tube, r_i [mm]	Diameter of Punch, D_p [mm]	Diameter of Die, D_d [mm]	Die Angle, [Degree]	Die End Radius [mm]
4	20	15	26.25	45	70	5
5	25	20	35	60	70	5
6	30	25	43.75	75	70	5
7	35	30	52.50	89.985	70	5

Table 3 Design Parameters for Steel Plate with L = 250 mm and t = 10 mm

r_o/t	Outer Radius of the Tube, r_o [mm]	Inner Radius of the Tube, r_i [mm]	Diameter of Punch, D_p [mm]	Diameter of Die, D_d [mm]	Die Angle, [Degree]	Die End Radius [mm]
6	30	25	43.75	149.975	70	10
7	35	30	52.50	179.970	70	10
8	40	35	61.25	209.965	70	10

Steel plate is modeled with PLANE 82 eight-node solid elements in plain strain state. In order to successfully model press-braking process, it was essential to define a contact between punch, plate and die. For this purpose, rigid to flexible surface to surface contact analyses have been performed by using targe 169 and Conta 172 elements. Two contact pairs have been created as shown in Fig.5.. First pair defines a contact between punch and upper surface of the plate which is a deformable body while the other pair is created between lower surface of the plate and die.

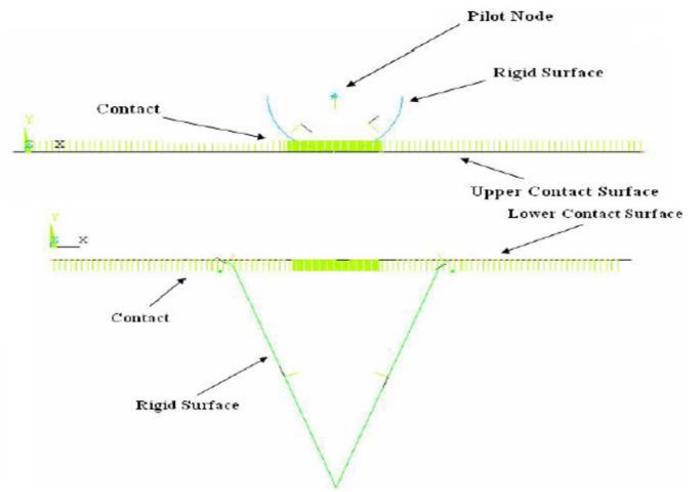


Figure 5 Contact Pair between Punch, Plate and Die

The plate is initially meshed with initial element size of 2 and later the mesh is refined in the region where residual stress distribution had to be captured as shown in figure(6). During these simulations the outer radius to thickness ratio (r_o/t) of 3, 4, 5, 6 and 7 have been achieved along with bent angles of 70^0, 80^0, 90^0, 100^0, 110^0, 120^0.

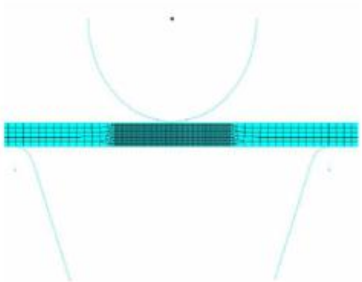


Figure 6 Mapped meshing for Steel plate modeled in ANSYS

Two load steps were required to carry out the whole analysis. In the first step, the load is applied by moving the punch down while in the other step the load is removed and punch moves back to its original position. As a result of this analysis, the magnitude at middle of the inner corner of the plate along with distribution of residual stress through thickness at middle of the plate is achieved.

4. Results and Discussions:

Extensive FE-simulations have been performed for S355 and S650 steels with different r_o/t ratios and bent angles. As a result of these analyses, the tangential residual stress obtained at the middle of the inside corners of the plate with different r_o/t ratios and bent angle as shown in figure (7). Since these simulations have been performed for two different types of steel i.e. S355 and S650 hence the results have been discussed separately for them.

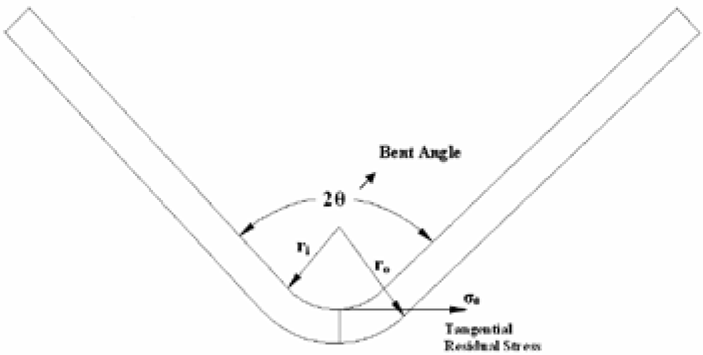


Figure 7 Cold bent plate showing bent anlge, inner and outer radius and point where tangentail stress is to be determined

4.1 Cold forming of S355 and S650 Steel Plates:

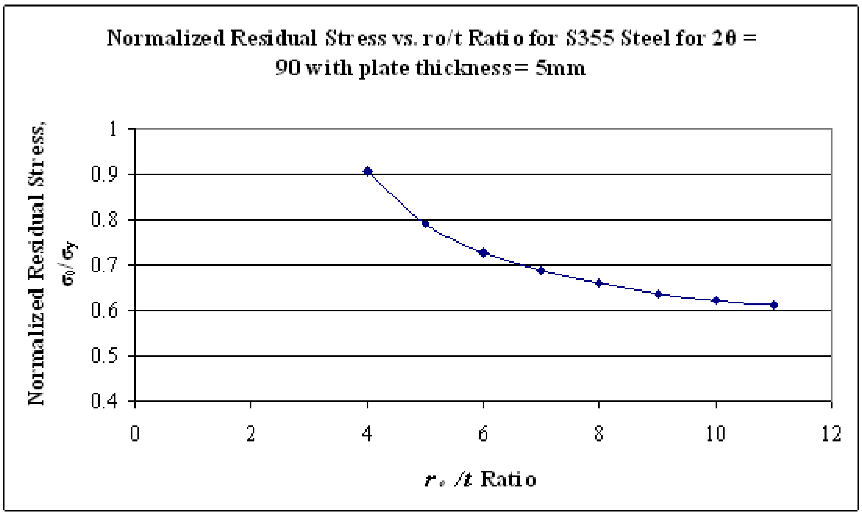
4.1.1 Magnitude of Residual Stress:

The magnitudes of residual stress obtained from analyses after spring back are listed in table 4.

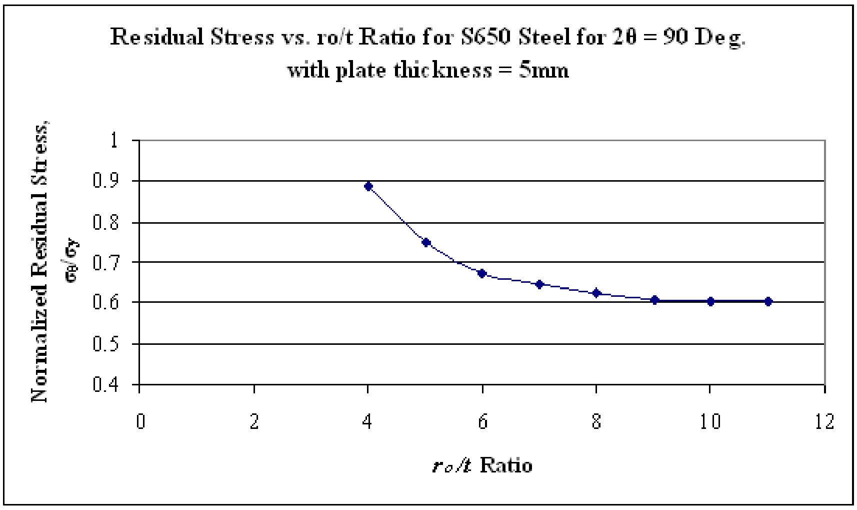
Table 4 Magnitudes of Residual Stress in S355 and S650 Steels

r_o/t	σ_θ, Residual Stress in S355 [MPa] Plate Thickness = 5 mm					σ_θ, Residual Stress in S650 [MPa] Plate Thickness = 5 mm				
	2θ 70^0	2θ 80^0	2θ 90^0	2θ 100^0	2θ 110^0	2θ 70^0	2θ 80^0	2θ 90^0	2θ 100^0	2θ 110^0
4	315	316	321	323	327	574	575	576	580	589
5	277	280	281	283	285	479	480	486	493	496
6	252	255	258	259	261	430	431	437	442	443
7	239	242	244	245	246	414	419	420	423	426
8	-	-	235	-	-	-	-	405	-	-
9	-	-	226	-	-	-	-	397	-	-
10	-	-	220	-	-	-	-	394	-	-
11	-	-	217	-	-	-	-	394	-	-
	Plate Thickness = 10 mm					Plate Thickness = 10 mm				
	2θ 70^0	2θ 80^0	2θ 90^0	2θ 100^0	2θ 110^0	2θ 70^0	2θ 80^0	2θ 90^0	2θ 100^0	2θ 110^0
6	-	-	341	-	-	-	-	666	-	-
7	-	-	313	-	-	-	-	598	-	-
8	-	-	300	-	-	-	-	543	-	-

Figure 8(a) and (b) shows a variation of residual stresses in S355 and S650 steel with r_o/t ratio that varies from 4 to 11 with bent angle of 90^0. It can be clearly seen that residual stresses are at highest level in lowest r_o/t ratio but as the r_o/t ratio is increased, the residual stresses begin to decrease until the fall in its value become very lower. From the plot, it can be seen that the residual stresses may become stabilized at some point if r_o/t value is further increased. It is also clear from the plot that the residual stresses obtained at middle of inner corner of the plate ranges from $0.60\sigma_y$ to $0.90\sigma_y$ for all r_o/t ratios. C.C. Weng and R.N White (Weng et al., 1990) also found that the measured residual stress at middle of inner corner of HY-80 steel plate ($\sigma_{y=}$593 MPa) of thickness 25.4 mm and 38.1 mm, ranged between 46% to 92% of the yield strength (σ_y) of the material. Quach and Teng (Quach et al., 2006) performed the finite element simulation using specimens (Weng and White, 1990). They found that the residual stresses at inner corner of the cold bent plate were 0.40 to 0.90 of the yield strength. Weng and White, (1990) and Quach et al. (2006) provide the results for cold bent steel plate which are in good agreement with finite element simulation results of S355 steel plates.



(a)

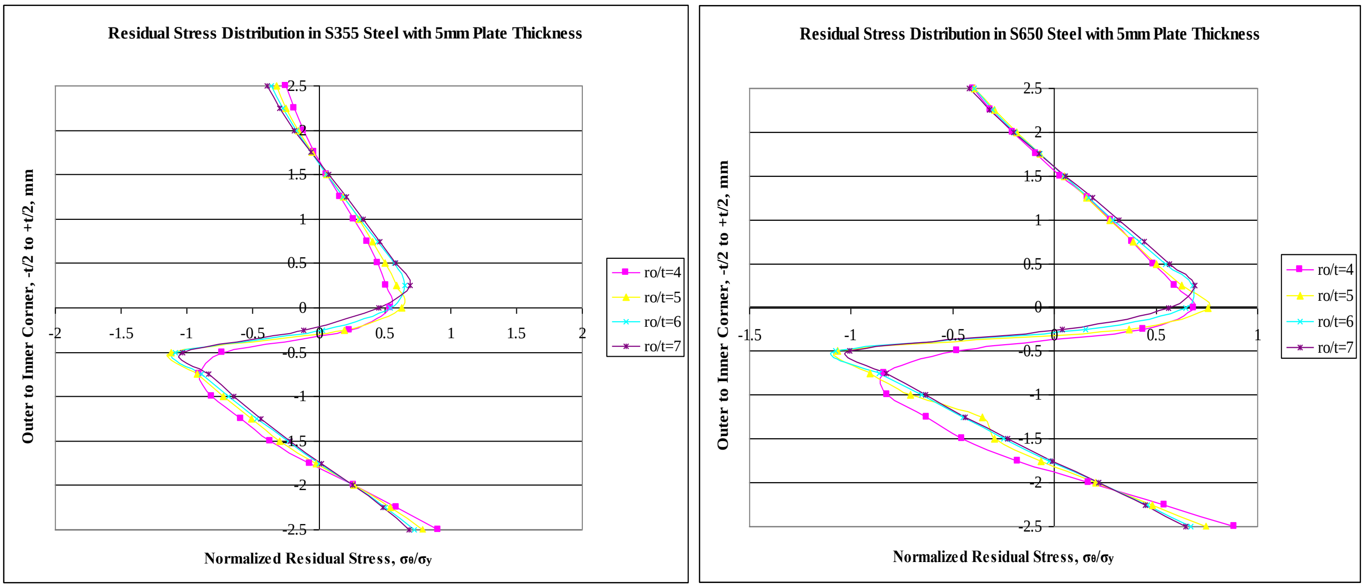


(b)

Figure 8 Normalized residual stress in S355 and S650 steels

4.1.2 Residual Stress Distribution:

As a result of these analyses, the residual stress distribution though thickness of the plate at the middle after spring back is also captured. The zig-zag type distribution of the residual stresses through the thickness of the bent plate is shown in figure 9(a) and (b) for all r_o/t ratios of 4,5,6 and 7 in case of bent angle of 90^0. The horizontal axis is obtained by dividing residual stresses through thickness of the plate to the yield strength of the material while the vertical axis represents thickness of the plate at corner region. The distribution of the residual stresses in all cases is in good agreement with figure 2 showing residual stress as a combination of loading and unloading stress. Cristopher D. Moen and Takeru Igusa (Cristopher et al., 2008) presented longitudinal residual stress distribution through the thickness of cold formed steel corners. Their residual stress distribution is in perfect agreement with figure 9(a) and (b) i.e. residual stresses are nonlinear through the thickness and its values are positive (tensile) at inner corner while negative (compressive) at outer corner. The shape of residual stresses presented in figure 9(a) and (b) is the similar to work presented by B. Kato and H. Aoki (Kato et al., 1978).

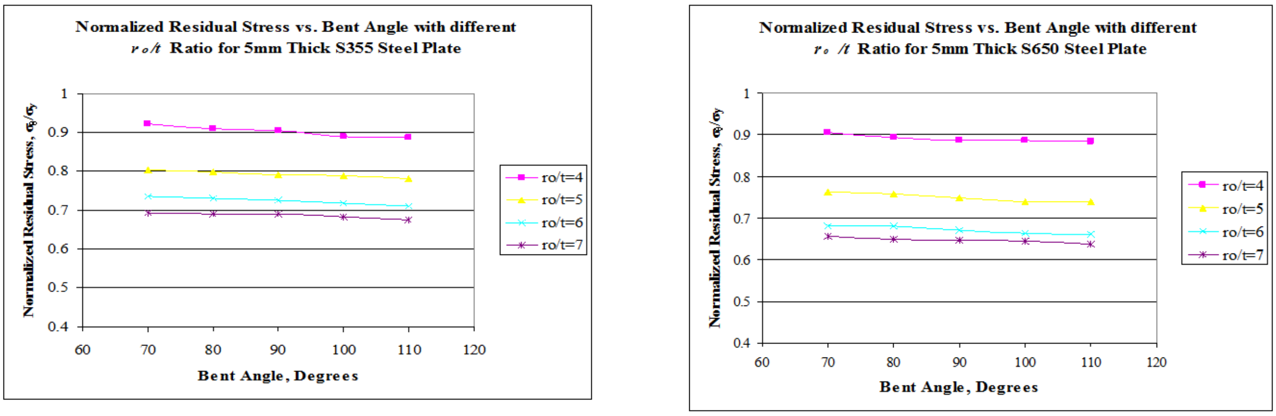


(a) **(b)**

Figure 9 Residual stress distributions in S355 and S650 steels

4.1.3 Effect of r_o/t Ratios and bent Angles:

Apart from finding magnitude and distribution of residual stress in the material, two other major aspects were investigated in detail i.e. the effect of r_o/t ratio and bent angles on residual stress. Figure 10 (a) and (b) shows the behavior of residual stresses in S355 and S650 steel with different r_o/t ratios and bent angles. It is clear from the plots that smaller corners have higher values of residual stress than larger corners. Similarly, small bent sections have higher magnitudes of residual stress than larger bent section but variation is not much significant. This information is in close agreement with Quach and Teng (Weng et al., 1990), who plotted effect of r_o/t ratio on residual stress distributions as a result of cold forming of steel with yield strength of 250, 350 and 450 MPa.



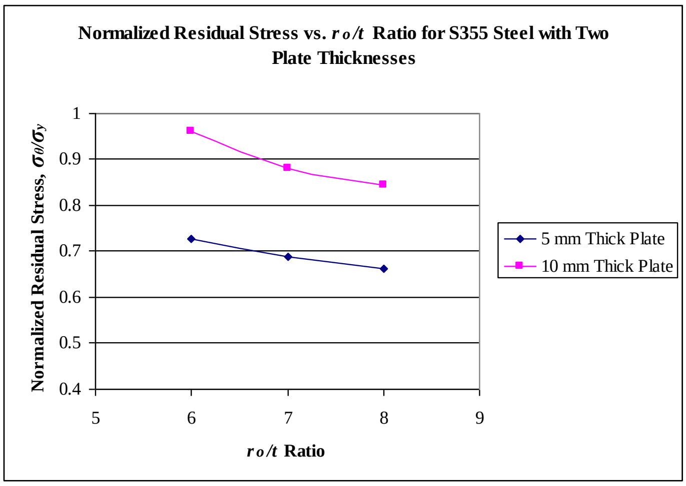
(a) **(b)**

Figure 10 Comparison of residual stress magnitiudes between different r_o/t ratios and bent angles for S355 and S650 steels

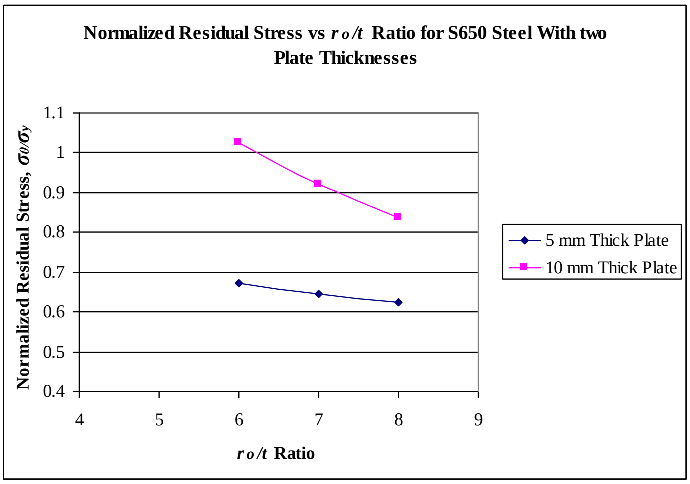
This information is also in very close agreement with C.C Weng and R.N White (Weng et al., 1990). According to them, the measured tension residual stress for HY-80 steel (σ_y = 593 MPa) of thickness 25.4 mm in the inside surface of the bend increases as the r_o/t ratio decreases, and decreases as the angle of bend increases. Further, W.M Quach and J.G. Teng (Quach et al., 2006) used finite element simulation for prediction of residual stresses in press-braked thin walled steel sections using same specimen used by C.C Weng (Weng et al., 1990), i.e. 1 inch (25.4 mm), HY-80 steel (σ_y = 593 MPa). It was clear from their results that the residual stresses at mid of the inner corners were higher with lower values of inner bent radius. Therefore, the information presented in figure 10 (a) and (b) is in good agreement with W.M Quach and J.G. Teng (Quach et al., 2006).

4.1.4 Effect of Thickness:

In order to find out the effect of thickness on residual stress in the material further simulations have been performed with r_o/t ratios of 6, 7 and 8 and bent angle of 90^0 with 10 mm thick plate and results have been compared with 5 mm thick plate. Figures 11 (a) and (b) represent the variation of residual stresses for S355 and S650 steel with two different thicknesses. It is clear from the figure that the residual stresses in 10 mm plate are about 22-25% for S355 and 25-35% for S650 higher as compared to 5 mm plate. It means that thickness does effect on residual stress i.e. higher residual stresses are achieved with thicker materials. C.C. Weng and R.N. White (Weng et al., 1990) presented residual stress values at inner corner of the bent plate in their work. These results were obtained by measuring the HY-80 (σ_y = 593 MPa) with three different thicknesses i.e. 25.4 mm, 38.1 mm, and 88.9 mm. From their measurement, it was clear that the residual stresses at inner corner of the bent plate were higher with 88.9 mm plate as compared to 38.1 mm plate. The residual stresses in case of 25.4 mm thick plates were lower than 88.9 mm and 38.1 mm. It clearly indicates that the plates with larger thicknesses have higher values of residual stresses.



(a)

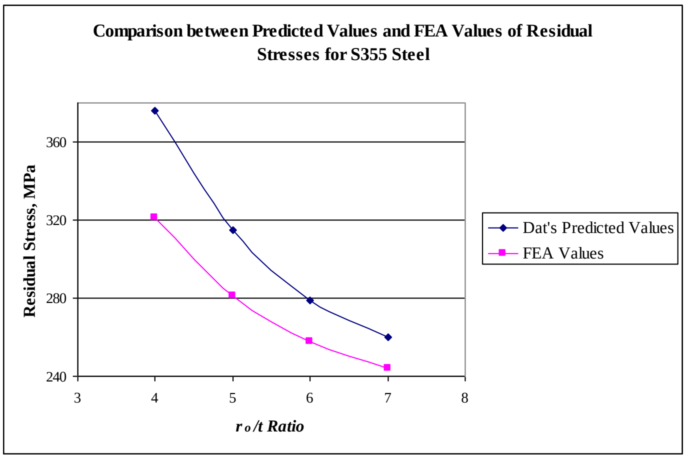


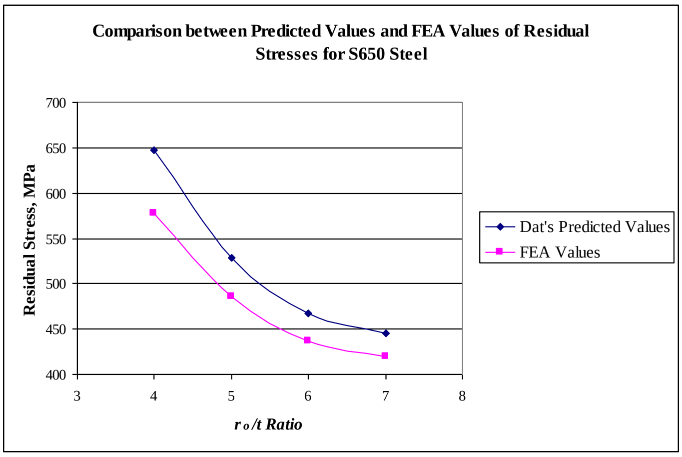
(b)

Figure 11 Comparison between residual stress magnitudes of 10 mm and 5 mm plates for S355 and S650 steels

4.1.5 Comparison of FEA results with Dat's Model:

Dat's presented model equations for determination of residual stresses in cold formed bent plates (Dat et al., 1980). The principal advantage of Dat's equation is its simplicity, since it provides a closed-form solution. These equations have been utilized in this thesis to determine predicted values of residual stresses in order to compare them with the values obtained from FEA. Since, we are only interested in finding out the tangential residual stresses at inner corner of the tube, therefore, equations 2, 6, 8, and 10 has been used. The contact pressure p is obtained from ANSYS postprocessor, while a is the inner radius r_i and b is the outer radius r_o. The value of $2K$ is obtained using von mises criterion. The predicted values of residual stresses have been obtained using S355 and S650 steel with r_o/t ratio of 4, 5, 6, and 7 for bent angle of 90^0.





(a) **(b)**

Figure 12 Comparison between predicted and computed values of residual stresses for S355 and S650 steels

It is clear from the figures 12 (a) and (b) that the residual stresses obtained from Dat's model are little higher than FEA results. The variation in results ranges from 3% to 17%. The effect of strain-hardening is not taken into account in Dat's approach, and the bent portion is assumed to be fully plastic after bending (Dat et al., 1980). This assumption is not appropriate for S355 and S650 steel, since these materials show significant strain-hardening. This assumption is satisfactory for plates bent to 90^0 but not for plates bent to 70^0, 80^0, 110^0, and 120^0. Therefore, Dat's model has been used to compute the values of residual stresses only in case of plates with bent angle of 90^0.

5. Conclusions:

Extensive FE-simulations have been preformed to investigate the effect of residual stress in S355 and S650 structural steels. As result of these analyses, the magnitudes of residual stress at inner corner of the bent plate along with distribution of residual stress have been obtained with different r_o/t ratios and bent angles. Residual stress is found to be higher in small corners and less bent angles as compared with lager corner radii and big bent angles. Furthermore, steel plates with large thickness store more residual stresses than small thickness plates.

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