

## A new Methodology in Modeling Forest Fire Spread Using Cellular Automata

H. Gazmeh<sup>1\*</sup>, A. Alesheikh<sup>2</sup>, M. Karimi<sup>3</sup>

<sup>1, 2, 3</sup> Faculty of Geodesy and Geomatic Engineering, K. N. Toosi University of Technology

ISSN 2231-8844

© 2011 Design for Scientific Renaissance All rights reserved

### Article Info

Received: 24/3/2012

Accepted: 2/9/ 2012

Published online: 1/12/2012

### Abstract:

There is a wild vast forest fire every year, causing irrecoverable losses and changing the soil, living organisms and ecosystems of the region. Thus, modeling the spread mechanism of fires in forests seems necessary for disaster managements. This study analysis the most commonly used fire spread models and improves them by a more real circular model of fire spread. The model uses a cellular automata (CA) to demonstrate fire spreads on circular fronts. The cellular model is based on Moore neighborhood. Although numerous factors can affect the spread of fire, this study uses fire spread rate, wind speed, wind direction and topography. Several scenarios are used to assess the proposed model. The implementation confirmed the efficiency of the model in comparison with others. The proposed model can be applied in both homogeneous as well as heterogeneous forests.

*Keywords:* Forest fire spread; Cellular Automata; Circular fronts; Spread rate; Moore neighborhood.

### 1. Introduction

One of the most critical events in nature is forest fire. Each year, between 6 to 14 million hectares of forests in the world burned out. Consequently, such fires change in structure and composition of forests (Hernández et al. 2007).

During recent decades numerous studies have been done on forest fire modeling. Damages caused by this phenomenon encouraged scientific communities to do better and more accurate modeling (Hernández et al. 2007). Modeling fire could be classified into two categories: stochastic and deterministic (semi-empirical or mathematical) models. Fire spread modeling that use mathematical models can follow two approaches: Cellular Automata (CA) models and/or vector models (Hernández, et al., 2007). Based on a series of predetermined rules, most vector models determine the rate of fire spread in time and location (Finney, 1999). Vector models are more complex and use Huygen's principal to describe propagation

<sup>1</sup> Corresponding author, phone:+989353858494  
E-Mail: h.gazmeh@gmail.com

(E. Pastor, 2003; Weber, 1991a). A third category of forest fire models that has been identified is physical models (Weber, 1991a). Hernandez et al., (2007) worked on Karafyllidis mathematical model and improved it (Hernández, et al., 2007). Later, They propose a more realistic model based on hexagonal cells instead of square cells, this proposes a better prediction of fire fronts (Hernández et al., 2007).

On the other hand, according to the related forest fire variables, we could categorize forest fire models into, Surface forest fire spread, Crown fire and Ground models. Several models composed of a series of equations which relate environmental parameters to fire behavior variables have emerged from the activity over these years (E. Pastor, 2003). Expressions for the rate of spread, fire fronts and fuel consumption are obtained from physical fuels features and also could be obtained from weather conditions. In Catchpole and De Mestre (Catchpole T, 1986), Weber (Weber, 1991b) and Perry (Perry, 1998) revisions of existing surface fire behavior models that are classified as theoretical, empirical and semi-empirical can be found (E. Pastor, 2003). Crown fire modeling are based on two issues: crown fire transition conditions (Molchanov, 1957), and crown fire behavior variables. Van Wagner (CE. Van Wagner, 1989) developed a semi-empirical model to obtaining the crown forest fire spread rate in Canadian conifer forests. Recent, there have been works in crown fire initiation. The most known are the Grishin (Grishin AM. Tomsk, 1997) model, due to its confirm theoretical treatment, the Albini model (Albini, 1996), which was calibrated and tested in the most complex and documented experimental research programs (Alexander, Stocks, Wotton, & Lanoville, 1998) and the models developed by the Forestry Canada Fire Danger Group (Group, 1992) and by Finney (Finney, 1993). The Van Wagner models (Van Wagner, 1977; CE. Van Wagner, 1989), and Finney (Finney MA. , 1993) uses the Rothermel (Rothermel & . 1991) model. Ground fires are characterized by burning without flame and by spreading very slowly; however they are very dangerous because they consume the organic layer of the soil.

There are many models like Tarifa et al. (Tarifa CS, 1965) Semi-empirical United States Muraszew and Fedele models (Muraszew A, 1976), Theoretical United States Albini (Albini, 1983) and Ellis (Ellis, 2000) Theoretical Australia Woycheese et al.

The purpose of this paper is to provide a model that resembles the reality better and with higher efficiency. This base idea is borrowed from Karafyllidis work. The transfer of fractional burned cells is discussed in more details. By using two-dimensional cellular automata, we achieve our goals. Cellular automata can predict complex phenomenon. Using the CA can simplify modeling phenomenon like fire, and urban development and so on (Chopard & Droz, 1998; Preston, Duff, & M.J.B, 1984). Several methods for modeling the progress in change detection and cryptography are based on cellular automata (Hernández, et al., 2007). In recent years, the study on integrating cellular automata with artificial intelligent such as Genetic algorithm, Fuzzy logic are also considered (Mohammad, Alesheikh, Behzadi, & Salehi, 2011; Vakalis, Sarimveis, Kiranoudis, Alexandridis, & Bafas, 2004). Although the above models are efficient for the modeling of fire, but other methods have also been successful in this field (Carvalho, Carola, & Tome; Stepanov & Smith, 2012).

The rest of the paper is as follows: In Section 2, the theory of cellular automata and its structure is presented. Karafyllidis model (Karafyllidis & Thanailakis, 1997) and its

modification (Hernández, et al., 2007; Hernández Encinas, et al., 2007) is described in Section 3. Our proposed model has been discussed in detail in Section 3. To test the model over several scenarios and compare the results with Karafyllidis model, the outcomes are presented in Section 4. Conclusions and further work will be in Section 5.

## 2. Cellular Automata

A cellular automata is a discrete dynamic system. The system consists of a grid network, which is usually a square grid in some papers also hexagonal cells have been used (Hernández, et al., 2007). Each of these components is called a cell. Each cell has a value at a time. The state of cells in discrete time steps, are updated based on a series of transition rules. These rules are defined based on neighboring cells. With this interpretation, a cellular automaton is defined as:

$$S_{(i,j)}^{(t+1)} = f(S_{(i,j)}^{(t)}, S_{\varphi(i,j)}^{(t)}) \quad (1)$$

This function shows that the state of  $(i, j)$  at time  $t + 1$ , is a function of its own state ( $S_{(i,j)}^{(t)}$ ) and the state of neighboring cells ( $S_{\varphi(i,j)}^{(t)}$ ) at time  $t$ . In general cases, a cellular automaton is a function of four factors:

$$Q = f(C, S, N, f) \quad (2)$$

Cellular space, which may indicate that  $C$  is  $n*n$  grid:

$$C = \{(i, j), 1 \leq i \leq n, 1 \leq j \leq n\} \quad (3)$$

State of each cell ( $S$ ), indicating the state of the cell in time step  $t(S_{(i,j)}^{(t)})$ . A set of  $S$  can be finite or infinite. The state of each cell in every time is obtained from set of  $S$ .

$$S = \{1, 2, \dots, n\} \quad (4)$$

Where  $n$  is a finite number.

By considering the central cell  $(i, j)$ , some kinds of neighborhood can be defined. The simplest type of neighborhood is the Von Neumann neighborhood. Another type of neighborhood is Moore's neighborhood, which is used in this paper (Fig. 1). Considering the Moore, two types of neighborhood can be considered. These are adjacent and diagonal neighbors (Alexandridis, Vakalis, Siettos, & Bafas, 2008; Hernández, et al., 2007).

$(i-1,j-1)$	$(i-1,j)$	$(i-1,j+1)$
$(i,j-1)$	$(i,j)$	$(i,j+1)$
$(i+1,j-1)$	$(i+1,j)$	$(i+1,j+1)$

Fig. 1. The Moore neighborhood

$$\begin{cases} V_{(i,j)}^{adj} = \{(i-1,j), (i,j-1), (i,j+1), (i+1,j)\} \\ V_{(i,j)}^{diag} = \{(i-1,j-1), (i-1,j+1), (i+1,j-1), (i+1,j+1)\} \end{cases} \quad (5)$$

Based on a series of transition rules in discrete time intervals cellular automata will be revolute. The state of cell  $(i, j)$  at time  $t + 1$  depends on its state and 8 neighboring cells at time  $t$ .

$$S_{(i,j)}^{(t+1)} = f(S_{(i,j)}^{(t)}, S_{(i-1,j-1)}^{(t)}, S_{(i-1,j)}^{(t)}, S_{(i-1,j+1)}^{(t)}, S_{(i,j-1)}^{(t)}, S_{(i,j+1)}^{(t)}, S_{(i+1,j-1)}^{(t)}, S_{(i+1,j)}^{(t)}, S_{(i+1,j+1)}^{(t)}) \quad (6)$$

The state of each cell at each time step is calculated based on Eq. 3 and the states of all cells in the time step  $t$  are shown by  $Q^{(t)}$ :

$$Q^{(t)} = \begin{pmatrix} S_{(1,1)}^{(t)} & \dots & S_{(1,n)}^{(t)} \\ \vdots & \ddots & \vdots \\ S_{(n,1)}^{(t)} & \dots & S_{(n,n)}^{(t)} \end{pmatrix} \quad (7)$$

This matrix describes the structure of cellular automata in each epoch of time then  $Q^{(0)}$  shows the initial state of cellular automata.

### 3. The CA-Model for fire fronts simulation

#### 3.1. Description of the basic model (Karafyllidis model)

The main model is presented by using cellular automata with a finite set of states. For simplicity in programming the boundary conditions also should be considered.

If the forest grid consists of hypothetical square cells with side length  $x$ , then the state of each cell is defined by the following equation (Karafyllidis & Thanailakis, 1997):

$$S_{(i,j)}^{(t)} = \frac{\text{Burned out area in cell } (i,j)}{\text{Total area of cell } (i,j)} \quad (8)$$

Then  $S_{(i,j)}^{(t)}$  has a series of continuous values from 0 to 1.  $S_{(i,j)}^{(t)} = 0$ , means that the cell  $(i, j)$  is still unburned at time  $t$  and  $S_{(i,j)}^{(t)} = 1$  means the cells completely burned out at time  $t$ .

$0 < S_{(i,j)}^{(t)} < 1$  It means that the cell  $(i, j)$  is partially burned out.

Each cell in the cellular space has a series of characteristics that indicate the value of the cell. For each cell, fire spread rate ( $r_{ij}$ ), topography ( $h_{ij}$ ) and wind speed and direction ( $w_{ij}$ ) is considered.  $r_{ij}$  indicates the rate of fire spread. In our case study the rate of fire spread is used similar to (McRaei, 1989). The unit of rate of fire spread is  $m/s$ .

In a flat homogeneous forest with no wind conditions, the required time for fire to spread to adjacent cell from a burned out cell is as below (Fig. 2):

$$\tilde{t} = \frac{x}{r_{(i,j)}} \tag{9}$$

If only one cell is burned, in this case, the time required to transfer the fire to one diagonal cell is calculated from the following equation (Fig. 2).

$$\tilde{t} = \frac{\sqrt{2}x}{r_{(i,j)}} \tag{10}$$

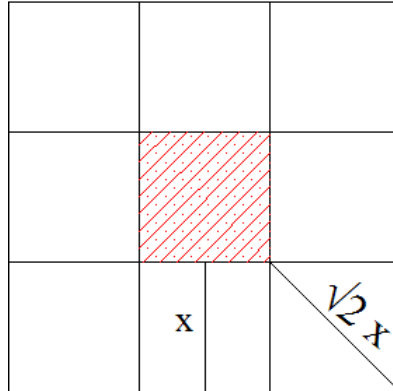


Fig. 2. Distance from a central cell to its adjacent and diagonal cells

Suppose that a cell is burned out completely in time  $t$  ( $S_{(i,j)}^{(t)} = 1$ ), as shown in (Fig. 3). Then, in the next time step ( $t + 1$ ) the adjacent cells will completely burned out:

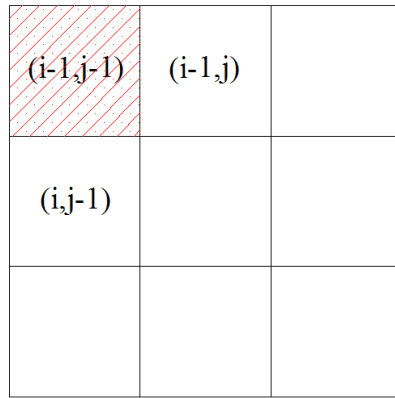


Fig. 3. The Propagation to adjacent cells

$$S_{(i-1,j)}^{(t+1)} = S_{(i,j-1)}^{(t+1)} = 1$$

Fire spread at a time steps depends on cell sizes, and so, for diagonal cell  $(i,j)$  fire spreads similar to Figure 4. It means  $S_{(i,j)}^{(t)} = \gamma$  where  $\gamma < 1$ . Karafyllidis assumed that fire spreads linearly from central cell to diagonal cells(Karafyllidis & Thanailakis, 1997). With simple calculations, the  $\gamma$  value for diagonal cells can be obtained as:

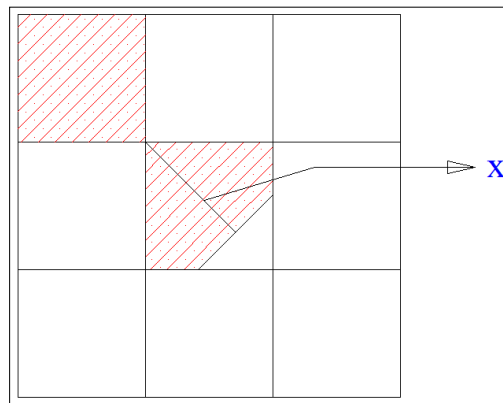


Fig. 4.Linear propagation to diagonal cell

$$S_{(i,j)}^{(t)} = \frac{x^2 - ((\sqrt{2} - 1)^2 x^2)}{x^2} = 2(\sqrt{2} - 1) \cong 0.83$$

According these cases, the cellular automata local transition rule is:

$$S_{(i,j)}^{(t+1)} = S_{(i,j)}^{(t)} + \left( S_{(i-1,j)}^{(t)} + S_{(i,j-1)}^{(t)} + S_{(i,j+1)}^{(t)} + S_{(i+1,j)}^{(t)} \right) + 0.83 * \left( S_{(i-1,j-1)}^{(t)} + S_{(i-1,j+1)}^{(t)} + S_{(i+1,j-1)}^{(t)} + S_{(i+1,j+1)}^{(t)} \right) \tag{11}$$

Considering this transition rule, the results were almost satisfactory. However (Hernández, et al., 2007), calculated another coefficient for the spread of fire from the middle cell to

diagonal cells. He assumed that the fire spreads from the central cell to diagonal cells radially not linearly (Fig. 5).

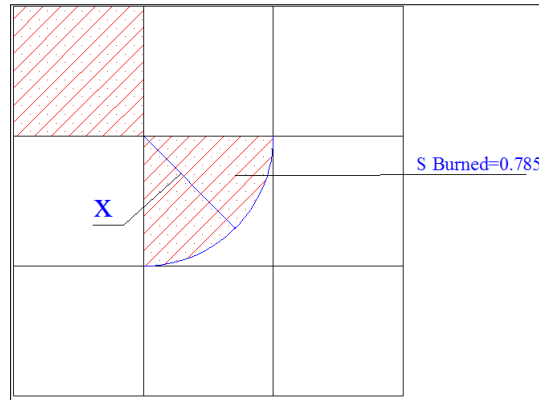


Fig.5. Nonlinear propagation to diagonal cell

With this approach, if only one cell at time step  $t$  is completely burned out, then the diagonal cells burn partially at time  $t+1$ . After this stage, the state of the diagonal cell in time  $t+1$  will be equal to:

$$S_{(i,j)}^{(t+1)} = \frac{\pi x^2}{4 x^2} = \frac{\pi}{4} \cong 0.785$$

With these calculations, the local translation rule will change to:

$$S_{(i,j)}^{(t+1)} = S_{(i,j)}^{(t)} + (S_{(i-1,j)}^{(t)} + S_{(i,j-1)}^{(t)} + S_{(i,j+1)}^{(t)} + S_{(i+1,j)}^{(t)}) + 0.785 * (S_{(i-1,j-1)}^{(t)} + S_{(i-1,j+1)}^{(t)} + S_{(i+1,j-1)}^{(t)} + S_{(i+1,j+1)}^{(t)}) \quad (12)$$

### 3.2. The proposed circular CA-Model

Our proposed model is based on the reality that the fire spreads in a circular manner. In this methodology fire spreads in circular shape. This kind of propagation when there is a flat homogenous forest with no wind blowing confirmed by other studies (Byram, 1959; Karafyllidis & Thanailakis, 1997; Luke & McArthur, 1978). So, when the state of the starting point of combustion is 1, then the fire spread will be as nearly concentric circles around this point. At the time,  $t = 0$  the ignition point is  $S_{(i,j)}^{(t=0)} = 1$ , at the moment,  $t = 1$  the state of neighbor cells is calculated based on Fig. 6:

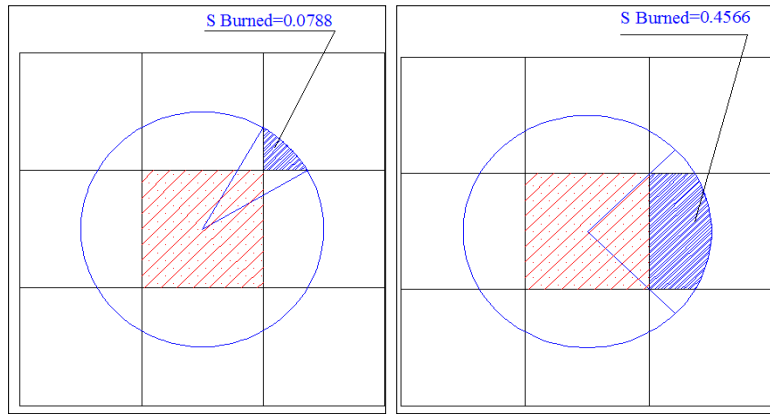


Fig. 6. Circular propagation to adjacent and diagonal cells

In this situation, even adjacent cells do not completely burn out at time  $t=I$  and all adjacent and diagonal cells burn out partially.

$$\begin{cases} s_{(i,j)}^{(adj)} = \alpha, & \alpha < 1 \\ s_{(i,j)}^{(diag)} = \beta, & \beta < 1 \end{cases}$$

After doing simple calculations based on Fig. 6

$$\alpha = 0.4566 \text{ And } \beta = 0.0788$$

Therefore, the proposed CA local transition rule will change as:

$$S_{(i,j)}^{(t+1)} = S_{(i,j)}^{(t)} + 0.4566 * (S_{(i-1,j)}^{(t)} + S_{(i,j-1)}^{(t)} + S_{(i,j+1)}^{(t)} + S_{(i+1,j)}^{(t)}) + 0.0788 * (S_{(i-1,j-1)}^{(t)} + S_{(i-1,j+1)}^{(t)} + S_{(i+1,j-1)}^{(t)} + S_{(i+1,j+1)}^{(t)}) \quad (13)$$

State of cells at different time steps could be larger than 1, so for the simplification purposes the following function is used:

$$\begin{cases} 0 & ; \text{if } 0 \leq s_{(ij)}^{(t)} < 1 \\ 1 & ; \text{if } s_{(ij)}^{(t)} \geq 1 \end{cases}$$

In the proposed model, spread rate of fire ( $r_{ij}$ ), wind speed and direction ( $w_{ij}$ ) and topography ( $h_{ij}$ ) is considered. In general:

$$S_{(i,j)}^{(t+1)} = S_{(i,j)}^{(t)} + 0.45 * \sum_{(\alpha,\beta) \in V^{adj}} w_{(i+\alpha,j+\beta)} * h_{(i+\alpha,j+\beta)} * S_{(i+\alpha,j+\beta)}^{(t)} + 0.0788 * \sum_{(\alpha,\beta) \in V^{diag}} w_{(i+\alpha,j+\beta)} * h_{(i+\alpha,j+\beta)} * S_{(i+\alpha,j+\beta)}^{(t)} \quad (14)$$



If there were some kinds of vegetation types, with the assumption that each cell could have different rate of fire spread, it must incorporate this factor on (Eq. 10). Just  $(r_{ij} / r)$  must be multiplied by Eq.10 that the  $r_{ij}$  is fire spread rate for each cell and  $r$  is the maximum rate.

#### 4. Testing the proposed circular CA-Model in hypothetical forest

A good model must have enough efficiency and should pass several tests. Various scenarios are discussed in here. Programming environment of this study is MATLAB 2009 b.

##### 4.1. Homogeneous forest

##### 4.1.1. No wind and no topography:

In homogeneous forests and no wind conditions and no topography, it is expected that fire fronts be circular from the ignition point (Fig. 7). In this case Karafyllidis model (Fig. 7. (a)) and its modification (Hernández, et al., 2007)(Fig. 7. (b)) and our proposed model (Fig. 7. (c)) are presented. In both Karafyllidis and its modification fire spreads to adjacent cells linearly as could be seen in outputs, but in this model propagation to both adjacent and diagonal cells is circular. It is obvious that our model is more realistic. For all scenarios, ignition point is central point and the units in both axes are arbitrary.

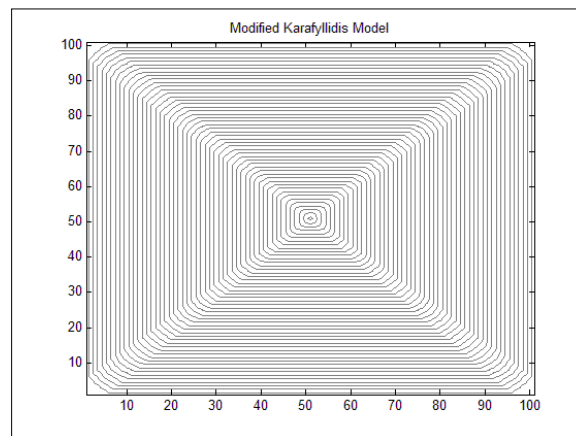


Fig.7. (a).Fire fronts in flat homogeneous forest with no wind (Karafyllidis model)

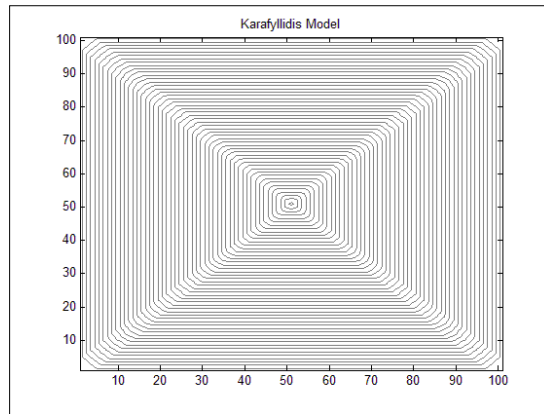


Fig. 7. (b). Fire fronts in flat homogeneous forest with no wind (Modified Karafyllidis model)

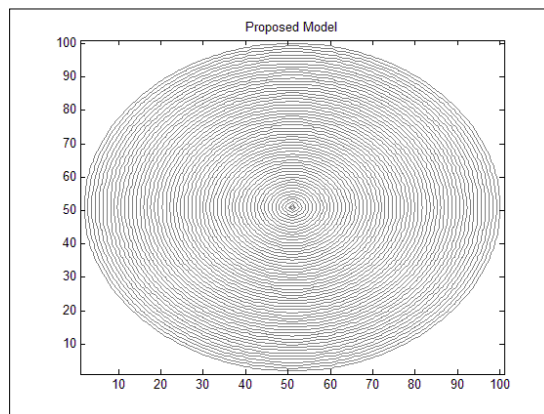


Fig. 7. (c). Fire fronts in flat homogeneous forest with no wind (Proposed model)

#### 4.1.2. West wind and no topography:

When the wind is blowing, fire spreads more quickly in the direction of blowing. In contrast, in the opposite direction, the speed decreases (Fig. 8).

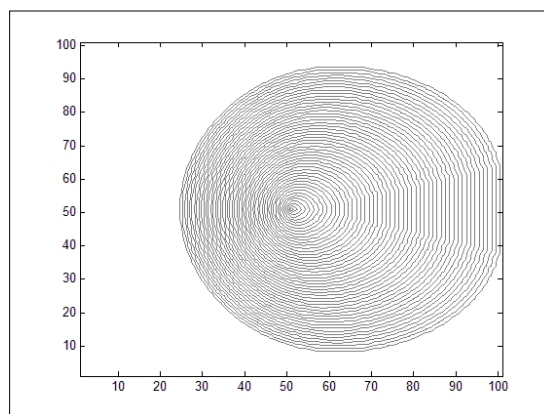


Fig. 8. Fire front in flat homogeneous forest and wind blow from west to east (North is top)

#### 4.1.3. Topography and no wind

It is clear that in upward slope the fire spreads more quickly. In such situations, the angle between flame and terrain reduces. Hence, transfer more heat to upward cells and consequently fire spread rate increase in positive slopes (Fig. 9). Suppose that there is topography as below (in this range we have a slope upward):

$$\begin{cases} 1 \leq i \leq 50 \\ 1 \leq j \leq 50 \end{cases}$$

To see the impact of topography in simulation, we use topography matrix as below (These values are arbitrary just to show the impact of topography):

$$\text{wind matrix} = \begin{pmatrix} 3 & 1 & 0.2 \\ 3 & 1 & 0.2 \\ 3 & 1 & 0.2 \end{pmatrix}$$

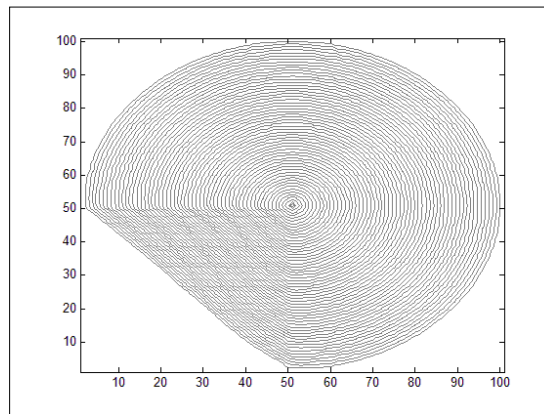


Fig. 9. Fire fronts in homogeneous forest with topography and no wind blowing

#### 4.2. Heterogeneous forests

In heterogeneous forest case, suppose for each cell, there is a rate of fire spread. Suppose there are various kinds of vegetation as below.

$$r_{ij} = \begin{cases} 1 & \text{for } 1 \leq i \leq 50, 1 \leq j \leq 50 \\ 3 & \text{for } 1 \leq i \leq 100, 50 < j \leq 100 \\ 4.5 & \text{for } 50 < i \leq 100, 1 \leq j < 50 \end{cases}$$

As seen, where  $r$  values are larger, fire fronts spread faster (Fig. 10).

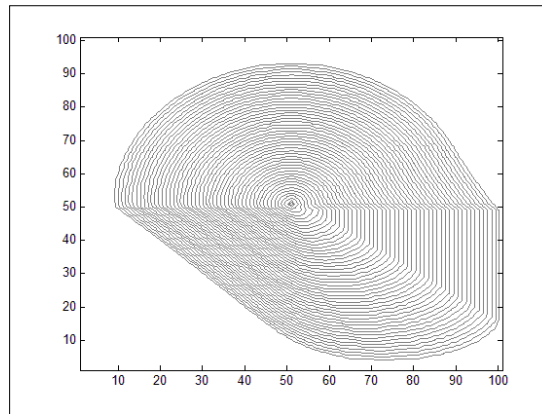


Fig. 10. Fire fronts in heterogeneous forest with no wind and no topography

#### 4.3. Incombustible areas

Our proposed model can easily incorporate incombustible cells in simulation. Values of incombustible cells in each time step remain 0 ( $s_{ij}^t = 0$ ). Incombustible cells cause to overthrow circular front and after several time steps become circular again (Fig. 11).

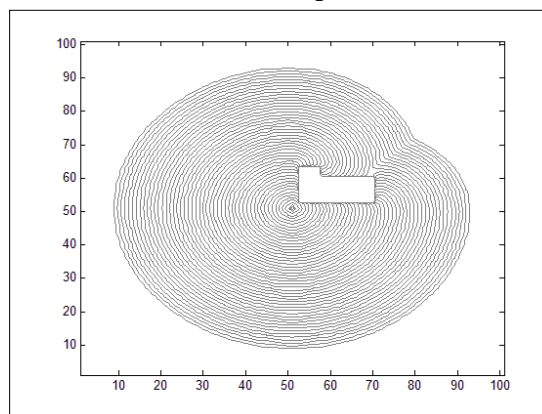


Fig. 11. Fire fronts in incombustible areas

### 5. Conclusions and further works

In this paper, a CA model for predicting forest fire front was proposed. This model actually is an improvement to Karafyllidis model that had linear spread for diagonal cells from center of cell. The basis of this work is on reality that fire spreads in circular fronts in flat homogenous forest with no wind condition, which is true for both adjacent and diagonal cells. Under this condition and based on calculations asserted that after initial state, even adjacent cells are not completely burned out, but partially ( $\alpha=0.4566$ ) and also diagonal cells are burned out partially ( $\beta=0.0788$ ). This model implemented in several scenarios, in the base scenario that there is no wind, no topography and forest is homogenous result must be a circular propagations that our model confirmed this. After this, model implemented in different scenarios like wind, topography and uneven fire spread rate for different vegetation types. Consequently, proposed model is more realistic and had a better agreement with literatures. This model could be used in both homogeneous and inhomogeneous forests and could apply wind conditions and topography. This model also supports spotting.

This work used Moore neighborhood, for further work, it is possible to expand the neighborhood radius. It is more useable for CA-models that work based on transferred heat from neighbor cells. Moreover, in closure, we could work on Calibration of CA-Models, by having historical burned area, it is good to discuss about calibration of CA time steps.

## References

- Albini, F. (1983). Potential spotting distance from wind-driven surface fires. USDA Forest Service, Res Pap INT-309.
- Albini, F. (1996). Iterative solution of the radiation transport equations governing spread of fire in wildland fuel. *Fizika Goreniya Zvryva*, 32(5), 71–82.
- Alexander, M., Stocks, B., Wotton, B., & Lanoville, R. (1998). An example of multi-faceted wildland fire research: the International Crown Fire Modelling Experiment. Paper presented at the Proceedings of the Third International Conference on Forest Fire Research, Portugal: University of Coimbra.
- Alexandridis, A., Vakalis, D., Siettos, C. I., & Bafas, G. V. (2008). A cellular automata model for forest fire spread prediction: The case of the wildfire that swept through Spetses Island in 1990. *Applied Mathematics and Computation*, 204(1), 191-201. doi: 10.1016/j.amc.2008.06.046
- Byram, G. M. (1959). Combustion of forest fuels. *Forest Fire- Control and Use*, 61-89.
- Carvalho, J. P., Carola, M., & Tome, J. A. B. Forest fire modelling using Rule-Bae fuzzy cognitive Map and Voronoi based Cellular Automata.
- Catchpole T, D. M. N. (1986). Physical models for a spreading line fire. 49(42):102–111.
- Chopard, B., & Droz, M. (1998). *Cellular Automata Modeling of Physical systems* Cambridge University Press.
- E. Pastor, L. Z., E. Planas, J. Arnaldos. (2003). Mathematical models and calculation systems for the study of wildland fire behaviour. *Progress in Energy adn Combustion Science*, 29, 139-153.
- Ellis, P. ( 2000). The aerodynamics and combustion characteristics of eucalypt bark, a firebrand study. PhD Thesis, Australian National University, Australia.
- Finney, M. (1993). Modeling the spread and behaviour of prescribed natural fires. Paper presented at the Proceedings of the 12th Conference on Fire and Forest Meteorology, Jekill Island, Georgia: Society of American Foresters.
- Finney, M. (1999). *Mechanistic modelling of landscape patterns*. Cambridge University Press, 186-209.
- Finney MA. , J. (1993). Modeling the spread and behaviour of prescribed natural fires. Paper presented at the Proceedings of the 12th Conference on Fire and Forest Meteorology, ekill Island, Georgia: Society of American Foresters.
- Grishin AM. Tomsk. (1997). A mathematical model of forest fires and new methods of fighting them. Publishing House of the Tomsk State University, Russia.
- Group, F. C. F. D. (1992). Development and structure of the Canadian forest fire behaviour prediction system. In C. D. o. Forestry (Ed.), (Vol. Inf Rep ST-X-3).

- Hernández, E., Hernández Encinas, L., Hoya White, S., Martín del Rey, A., & Rodríguez Sánchez, G. (2007). Simulation of forest fire fronts using cellular automata. *Advances in Engineering Software*, 38(6), 372-378. doi: 10.1016/j.advenzsoft.2006.09.002
- Hernández Encinas, L., Hoya White, S., Martín del Rey, A., & Rodríguez Sánchez, G. (2007). Modelling forest fire spread using hexagonal cellular automata. *Applied Mathematical Modelling*, 31(6), 1213-1227. doi: 10.1016/j.apm.2006.04.001
- Karafyllidis, I., & Thanailakis, A. (1997). A model for prediction forest fire spreading using cellular automata. [journal article]. *Ecological Modelling*, 99, 87-97.
- Luke, R. H., & McArthur, A. G. (1978). Bushfires in Australia. Paper presented at the Australian Government Publishing Service, Canberra.
- McRaei, R. (1989). Use of digital terrain data for calculating rates of fire spread with PREPLAN computer system. Paper presented at the Proc of the 8th biennial conference and bushfire dynamics workshop, Canberra.
- Mohammad, H. V., Alesheikh, A., Behzadi, S., & Salehi, S. (2011). Modeling the spread of spatio-temporal phenomena through the incorporation of ANFIS and genetically controlled cellular automata: a case study on forest fire. *International Journal of Digital Earth*. doi: 10.1080/17538947.2011.603366
- Molchanov, V. (1957). Conditions for the spread of crown fire in pine forest. *Lesnoe Khozydstvo*, 10, 50–63.
- Muraszew A, F. J. (1976). Statistical model for spot fire hazard. *Aerospace Report*, ATR-77(7588)-7581.
- Perry, G. (1998). Current approaches to modelling the spread of wildland fire: a review. *Prog Phys Geogr*, 22(2), 222–245.
- Preston, J., Duff, & M.J.B. (1984). *Modern Cellular Automata: Theory and Applications*. Plenum Press, New York.
- Rothermel, R., & . (1991). Predicting behaviour and size of crown fires in the northern Rocky Mountains. *USDA Forest Service, Gen Tech Rep INT-438*.
- Stepanov, A., & Smith, J. M. (2012). Modeling wildfire propagation with Delaunay triangulation and shortest. *European Journal of Operational Research*, 218, 775-788. doi: 10.1016/j.ejor.2011.11.031
- Tarifa CS, D. N. P., Moreno FG. (1965). On the flight paths and lifetimes of burning particles of wood. Paper presented at the enth International Symposium of Combustion, T, Pittsburgh, PA: The Combustion Institute.
- Vakalis, D., Sarimveis, H., Kiranoudis, C., Alexandridis, A., & Bafas, G. (2004). A GIS based operational system for wildland fire crisis management I. *Mathematical modelling and simulation*. *Applied Mathematical Modelling*. doi: 10.1016/j.apm.2003.10.005
- Van Wagner, C. (1977). Conditions for the start and spread of crown fire. *Can J For Res*, 7, 23–34.
- Van Wagner, C. (1989). Prediction of crown fire behaviour in conifer stands. Paper presented at the Proceedings at the 10th Conference on Fire and Forest Meteorology, Ottawa, Canada.

- Van Wagner, C. (1989). Prediction of crown fire behaviour in conifer stands. Paper presented at the Proceedings at the 10th Conference on Fire and Forest Meteorology, Ottawa, Canada.
- Weber, R. O. (1991a). Modelling fire spread through fuel beds. *Energy Combust. Sci.*, 17.
- Weber, R. O. (1991b). Modelling fire spread through fuel beds. *Energy Combust Sci*, 17, 62–82.