

Optimization of Concrete by Minimizing Void Volume in Aggregate Mixture System

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ABSTRACT

The behavior of concrete is affected by the size, distribution of the voids, the porosity and of the granulometry of the aggregate mixture. As a consequence it necessary for engineers to consider in detail particle packing concepts and their influence on the physical performance of concrete. The present study included determination of the fineness modulus, cement paste volume based on slump test and the properties of fresh and hardened concrete. It also comprised a study of the applicability of two theoretical models Toufar's and 4C-packing program for selecting suitable relative amounts of the concrete constituents, for obtaining a minimum void ratio. Comparison of the results was made from using these theoretical models literature-derived experimental data. The models gave similar results and suggest similar combinations of materials to give minimum void ratio. Optimization of the composition of the aggregate material in concrete is beneficial with respect to economy (low cement content), strength and durability. It was noted that minimizing the void ratio (V) tends to raise the stiffness and that the compressive strength (f_c') is closely related also to their molding (R) and fineness modules (FM). It was found that the compressive strength can be predicted by applying the formula: $f_c' = -2.1 - 63.8 V + 0.150 R + 10.4 FM$; $R^2=0.94$, and two other relationships related to slump data and cement content.

Keywords: Void ratio, Aggregate mixture, Fineness modulus, Cement paste volume, packing and Compressive strength.

1. Introduction

Mixture proportioning of cement-based materials (such as paste i.e. cement and water) and concrete can be described as a process of minimizing the volume of the voids in aggregate mixtures for obtaining the desired properties of fresh and hardened material.

Estimation of the packing density and the void ratio of concretes using particle packing models can provide tools to improve the performance of fresh and hardened materials by reducing the content of free water and cement and maximizing the amount of solids (Mohammed et al, 2012). However, the designer's problem is in proportioning the mix

constituents so as to provide a minimum void ratio and maximum packing density of the solid mixture components, while at the same time certifying sufficient workability.

The aim of this study is to utilize particle packing models, i.e. Toufar (Jones et al, 2002, Goltermann et al, 1997), and the Linear Packing Model (LPM), based on the Danish software 4C packing program (Senthil and Manu, 2003, Danish Technological Institute), for estimation of the void ratio of different aggregate. The study also included use of packing concepts for predicting the paste volume necessary to fill aggregate voids space. Finally, establishing theoretical models concerning the slump, cement content and compressive strength have been done.

2. Objectives and scope

Aggregate mixture composition is represented by a grain size distribution curve. The distribution and the density combine together to determine the most important physical properties of the aggregate and thereby also of the concrete, which can be of different types depending on the requirements (Traditional, SCC, HPC etc).

This study deals with:

- a. The aggregate being characterized by its void content density and fineness modules.
- b. Concrete workability related to remolding numbers.
- c. Slump cone test used to predict the required volume of cement paste.
- d. The concrete compressive strength.

These five issues are judged to give relevant information concerning the suitability of the aggregate composition, and are proposed to be used for optimizing the concrete properties.

The main objective of this paper is to investigate what is the most suitable aggregate composition which can be achieved using packing theory.

Questions to be answered are:

- a. How well do results from analytical and theoretical modeling agree with those from actual practical tests?
- b. How can the effect of different aggregate types be evaluated?
- c. Can one get desired concrete properties by determining the required cement paste volume through slump cone tests?

3. Literature review

Since 1907, Fuller and Thompson studied optimization of the properties of concrete by assessing the role of the size distribution of the aggregates (Mette and Ib, 1996). This was followed up by Suenson who applied early particle packing models around 1911 (Mette and Ib, 1996). Sedran et al. 1996, in applying the packing concept to design SCC concluded that the performance optimization of concrete is mainly a matter of improving the packing density of its granular skeleton (Wong, 2007). Several other investigators have applied and developed particle packing models such as Anderson, Powers, Aim and Goff, Toufar et al, Dewar, Yu et al and Delarrard (Wong, 2007, Delarrard, 1999; Joansen and Andersen, 1996; Goltermann et al, 1997; Mette and Ib, 1996; Dewar, 1999; Senthil and Manu, 2003; Delarrard and Sedran, 1994). Yu et al derived model based on linear packing.

A number of computer-based constituent proportioning methods had been developed,

such as the Danish software 4C-program (Danish Technological Institute), the Europack based on the modified Toufar model (Jones et al, 2002, Senthil and Manu, 2003, Quiroga and Fowler, 2003), the LCPC model based on DeLarrard models (Senthil and Manu, 2003, Sedran and Delarrard, 1999), and LISA based on Anderson's models (Senthil and Manu, 2003).

These programs are good opportunities for engineers to find an optimum combination of mix constituents for obtaining a minimum void ratio. By adopting one or several mathematical models for determining the void ratio and combining them, one can obtain the composition that gives the minimum porosity and permeability and maximum slump. One can hence optimize the properties of both freshly prepared and hardened concrete (Jones et al, 2002).

4. Models

4.1 Linear packing models

Stoval et al. 1986 and Delarrard 1999 have proven that there exists at least one dominated particle group in fully packed mixtures containing two or more particle groups (Stoval et al, 1986; Wong, 2007). However, the particles in such mixtures will be dispersed if they are not in close contact. The constituting equations of the linear packing model (Wong, 2007) are Eq. (1) and Eq. (2).

$$\omega = \omega_i = \frac{\eta_i}{1 - \sum_{j=1}^{i-1} [1 - \eta_i(1 + \lambda_{ji})] y_j - \sum_{j=i+1}^n (1 - \eta_j \lambda_{ji}) y_j} \quad (1)$$

$$\omega = \min_{1 \leq i \leq n} \omega_i \quad (2)$$

Where: ω : Actual packing density of the mixture; ω_i : Packing density of the particle group i ; η : Individual packing densities; λ : Particle interaction factor (loosening or wall effect factor); y : Individual solid particle to total solid particle Eq. (3); φ : solid volume of each particle group in a unit bulk volume.

$$y_i = \frac{\varphi_1}{\varphi_1 + \varphi_2} \quad (3)$$

4.2 Modified Toufar model

The Toufar model is usually applied in the form modified by Golterman et al (1997). The calculation of the packing density of particle combinations is dependent on the density and characteristic diameter of individual grains. The term "eigen-packing" is used for the packing degree factor of single components (Φ_1 and Φ_2) as shown in their paper.

4.3 Danish 4C software packing program (Danish Technological Institute)

The 4C-packing program can be used for calculating the packing density of any combination of solid constituents in concrete, i.e. aggregate, cement, etc. It is possible to get

the optimum concrete properties using a combination of a user-friendly program and the designer’s own experience (Mette and Gitte, 1993). This program can be utilized as a tool for comparing results of theoretical and practical tests and investigating the importance of the concrete constituent proportion, i.e. the concrete mix design (Claus et al, 2009).

5. Experimental programs

5.1 Materials

- Cement: CEM 11/A-LL 42, 5 R Skövde (Portland cement).
- Aggregate: Table 1 illustrates the fine and coarse constituents used in this study; Fig. 1 to 6 show the sieve analysis for the aggregate.
- Water: local water.

Table 1: Aggregate properties

Aggregate	Name and type	Fractionsizes (mm)		
		0-8	8-16	16-27
Aggregate 1	Rikstannatural	0-8	8-16	16-27
Aggregate 2	Rikstancrushed	0.5-1	2-4	8-11
Aggregate 3	Enhörna crushed	0-4	4-8	8-11
Aggregate 4	Källered crushed	0-0.5	0.5-1	1-2
Aggregate 5	Sand natural	0-2		
	Källered crushed	0-2	2-5	4-8
	Vikan crushed	11-16		

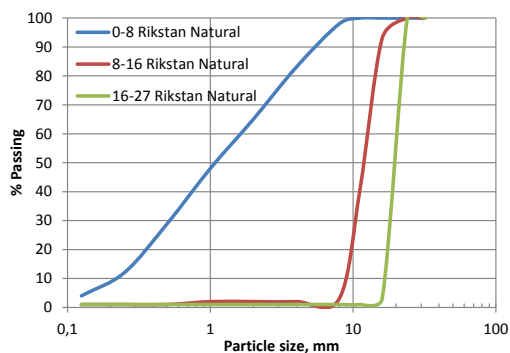


Fig.1

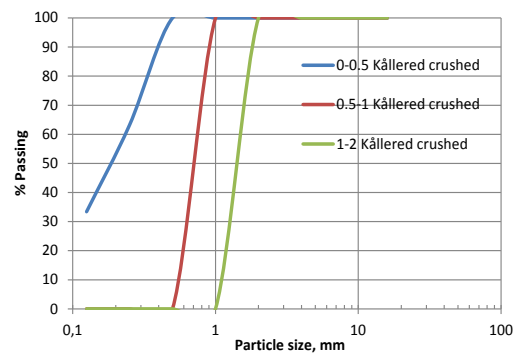


Fig. 4

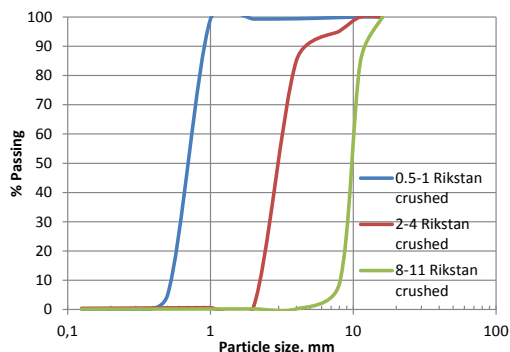


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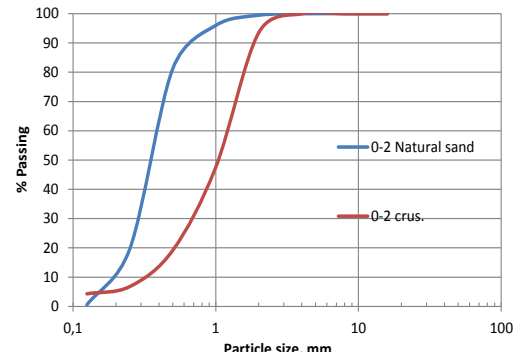


Fig.5)

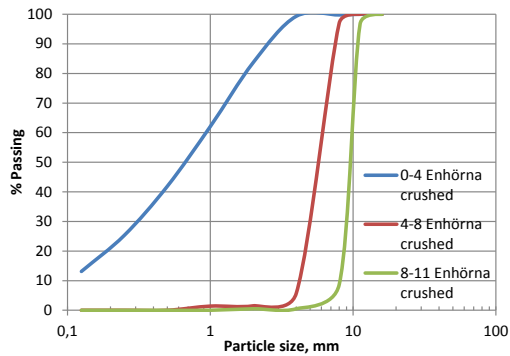


Fig.3

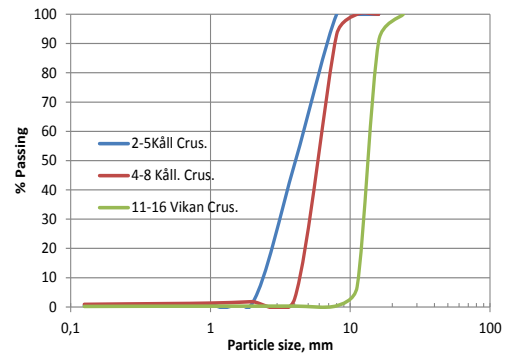


Fig.6

Fig.1-6. Sieve curves of tested aggregate.

5.2 Variables and mixture proportions

In the present laboratory study four different aggregate types (1 to 4 in Table 1) were investigated for determining how the mix void ratio depends on the packing concept (Kristoffer, 2005, Mohammed et al, 2012). The results were compared with those obtained theoretically. In addition, fresh concrete properties, workability and hardened concrete properties using different proportions of the aggregate types 5 and 3 were investigated.

5.3 Test methods

Most of the experimental work had been conducted by (Kristoffer, 2005). The test methods were as follows:

- Packing methods: Loosely packed material and aggregate poured into a bucket without further treatment. The bucket with the sample was weighed and the procedure repeated three times for each sample for certifying the density and void ratio.
- Fineness modulus: the fineness modulus was calculated according to the standardized Swedish method (Utsi, 2008). The fineness modulus represents the area above the grading curve and is equal to the summed amounts of materials that remained on the standard sieves sequence, (Fig. 1 to 6).
- Slump cone test: the slump cone test (defined by the Swedish standard SS 13 71 21) gives an approximate measure of the fluidity of freshly prepared concrete).
- Remolding number test: it is a measure of the workability of freshly prepared concrete and is defined by the Swedish standard SS 13 71 30.
- Compression test: the compressive strength was made according to the Swedish standard SSEN 12390-3 using 28 days old cubical specimen with 150 mm edge length.

6. Results and Discussion

6.1 Void ratio of aggregate mixture

Figs. 7a to d are graphical representations of the void ratio versus the aggregate mix proportions. The void volume being calculated from the packing degree tests was compared with results from the theoretical models. It was noticed that the 4C-Packing model gives results that are more similar to the results from Toufar's model when compared with those

obtained using the actual values as shown in Fig. 7a,b. However, Fig. 7c,d showed that 4C results overrate the results estimated by Toufar’s model.

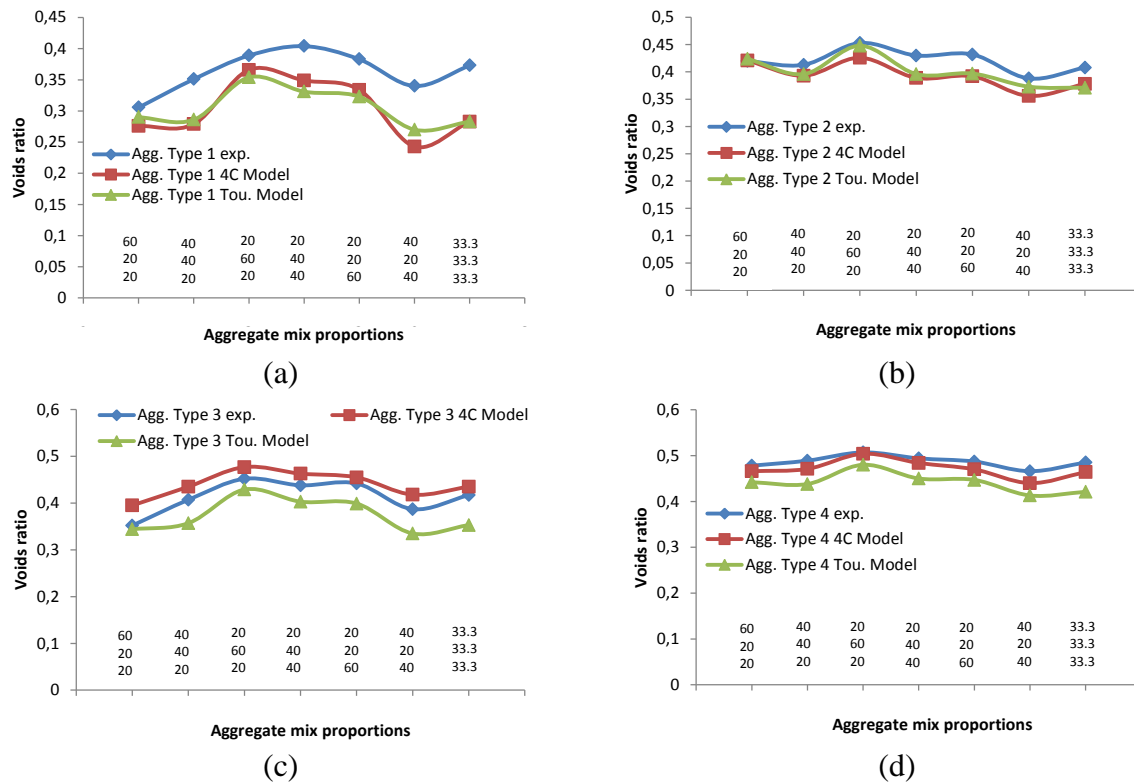


Fig. 7. Void ratio versus mix proportions

6.2 Minimum void aggregate

Figs. 8a,b and c show the calculated void ratio based on theoretical models compared with the results of the experimental packing of combinations of three fractions, coarse, intermediate and fine aggregates, and the suggested model of aggregate type 1. Fig. 8a shows the void ratios resulting from various combinations of the finer fraction (0-8 mm) and coarser aggregate fractions (8-16 and 16-27 mm), together with those calculated from the packing models. It can be seen that the Toufar’s model and 4C underestimated the void ratio. Dewar in 1999 based on this model suggested that the minimum void ratio requires finer fraction (Jones et al, 2002). Results based on theoretical models showed that similar results. In addition it suggests almost the same ratios for three size fractions for obtaining a minimum void ratio. The experimental also highlighted the important role of the fin fraction content (44-75 %) and indicated the necessity for making the mixture denser.

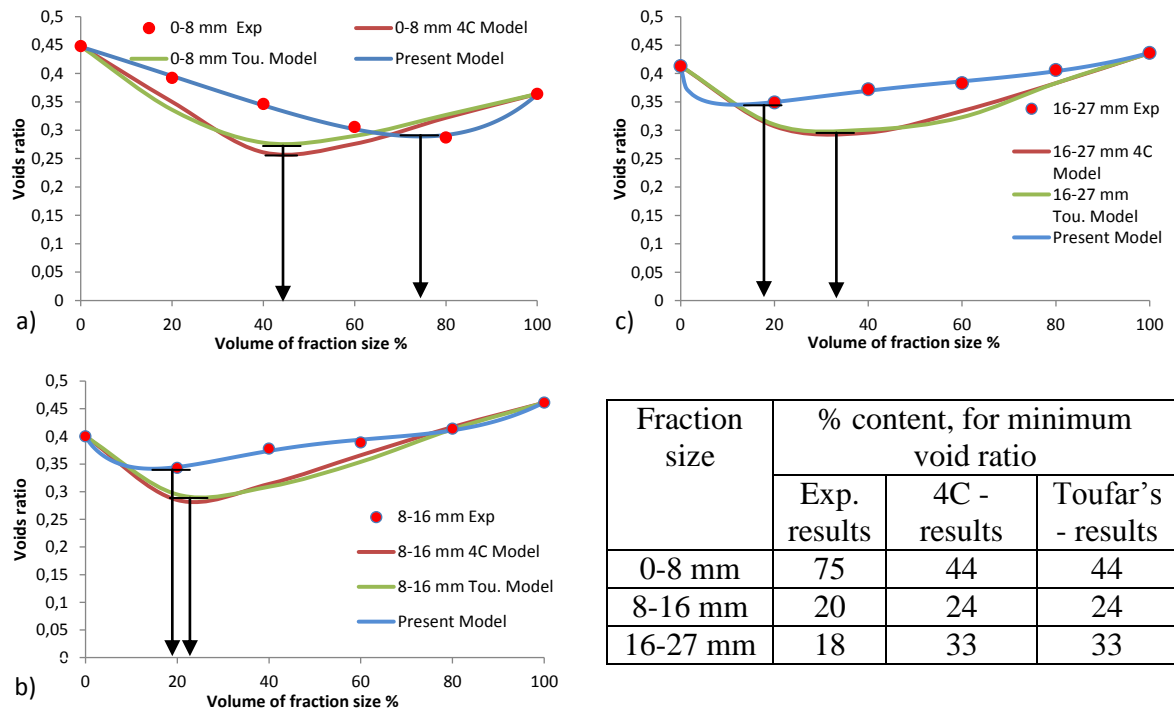
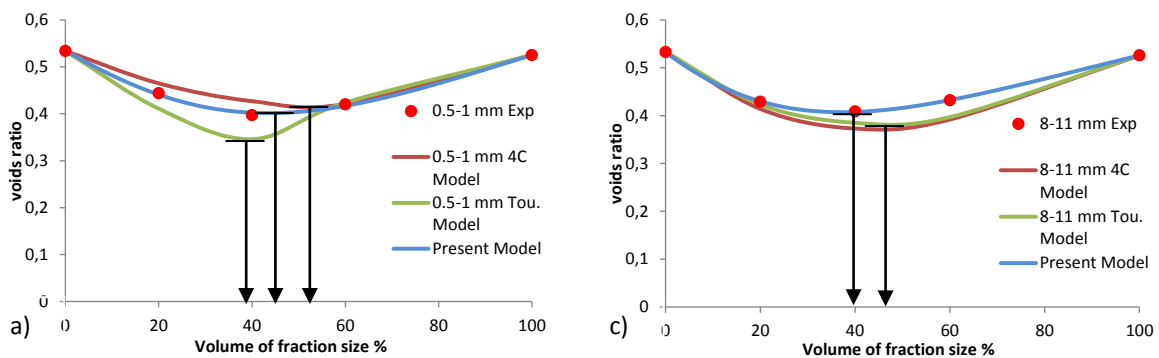
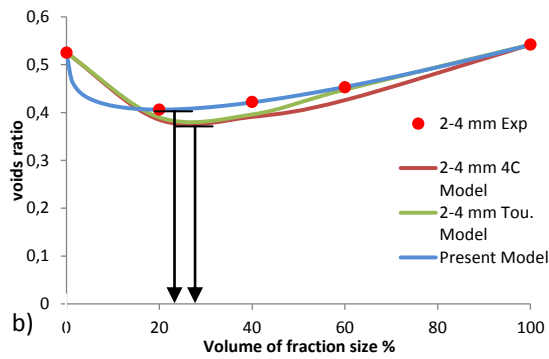


Fig. 8. Comparison of experimentally and theoretically derived void ratio as function of the volume of fraction size

Concerning aggregate type 2, results based on theoretical models (Toufar's & 4C) agreed well with the experimental data in Fig. 9. It can be seen that the Toufar's model underestimated the void ratio value while the 4C model overestimated it for the finer material. However, the voids ratios of the intermediate and coarser materials are overestimated when using Toufar's models Fig. 9b and c.

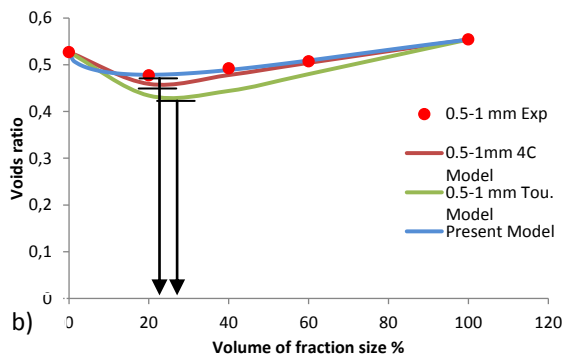
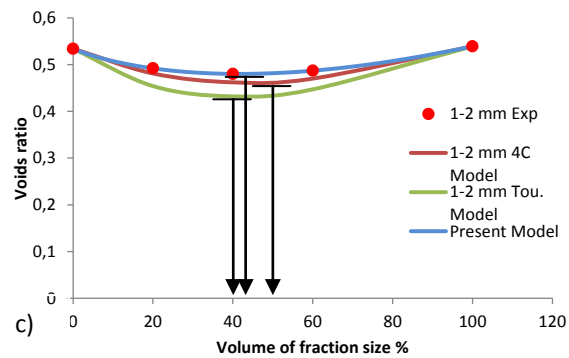
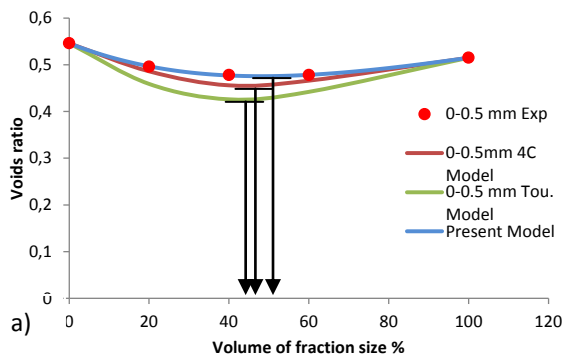




Fraction size	% content, for minimum void ratio		
	Exp. results	4C - results	Toufar's - results
0.5-1 mm	45	54	39
2-4 mm	24	28	28
8-11 mm	40	48	48

Fig. 9. Comparison of experimentally and theoretically derived void ratio as function of the volume of fraction size.

Finally, Fig. 10 shows the void ratios of various combinations of the aggregate type 4. It can be noticed that the Toufar's model and the 4C model agree very well with the test data in all the three figures. However, both underestimate the void ratio as compared with the actual values.



Fraction size	% content, for minimum void ratio		
	Exp. results	4C - results	Toufar's - results
0-0.5 mm	52	46	43
0.5-1 mm	24	24	27
1-2 mm	44	50	40

Fig. 10. Comparison of experimentally and theoretically derived void ratio as function of the volume of fraction size.

Comparison of theoretical and experimental results from the present study and the literature (Utsi, 2008) are illustrated in Fig. 11. It can be seen that all the models are reasonably accurate, because the difference between the measured and calculated mean differences void ratio does not exceed 2.9% to 3.5%. Some minor differences in function of the models can be identified as pointed out by (Jones et al, 2002).

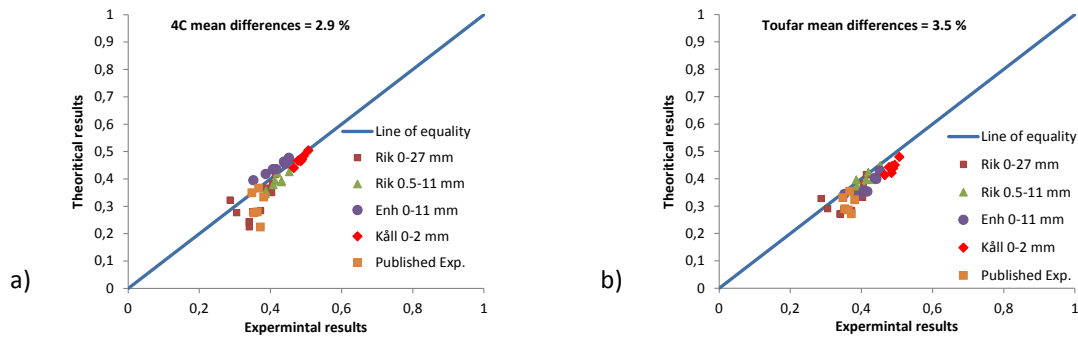


Fig. 11. Comparison of measured and calculated void ratio.

6.3 Fineness modulus and voids ratio

Aggregate specific surface is manifested by the fineness modulus. This surface determines the amount of cement paste needed to make the concrete workable. Minimizing voids by effective compaction reduces the amount of paste needed to fill them but the most important parameter for achieving a condition with minimum cement content is the specific surface, or in practice, the fineness modulus (Utsi, 2008). This is obvious for the aggregate tested in the present study as demonstrated by the relationship between the void ratio and the fineness modulus shown in Fig. 12a and b. These two figures illustrate the relationship between void ratio and fineness modulus grouped for 0-4 mm aggregate. One finds that the void ratio will decrease slightly as the fineness modulus increases until a minimum void ratio is attained after which the void ratio again starts to increase for an increased fineness modulus. Figs. 12a and b also show that almost the same void ratio can be reached for a range of fineness modules, which reflects the influence of the fine aggregate grains. The fineness modulus is correlated with the amount of concrete water and it is obvious that it is advantageous to choose a fineness modulus that is as high as possible. Finally, Fig. 13 gives an overview of all experimental data and those from other experimental work by (Utsi, 2008) as well as results from the theoretical model suggested by using the statistic software Minitab 16 and the regression analysis in Eq. 4.

$$V = 0.6237 - 0.1087 FM + 0.01196 FM^2 \tag{4}$$

Where V : Void ratio; FM : Fineness modulus.

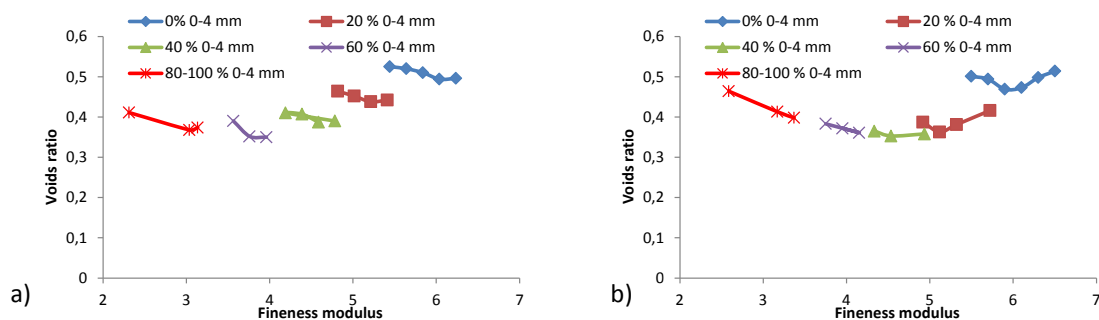


Fig. 12. Void ratio versus fineness modulus grouped by the fine aggregate content, a) present study aggregate type 3, Enhö.0-4 mm. b): published data (Utsi, 2008), 0-4 mm.

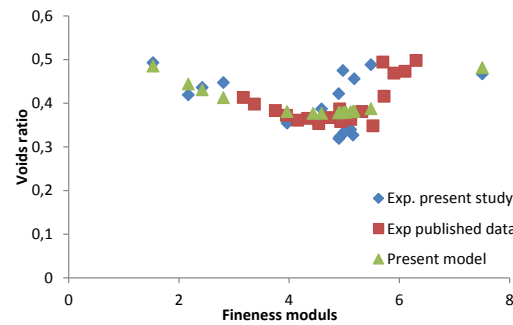


Fig. 13. Void ratio versus fineness modulus by all experimental aggregate type compared it with published data (Utsi, 2008) and present model suggest.

6.4 Fresh concrete

6.4.1 Concrete properties based on different aggregate types

The present study included preparation of concrete with different aggregate size fractions as defined in Tables 2 and 3. Fine natural sand (0-2 mm) was added as finest fraction for avoiding the problem with using fine crushed aggregate, which increases the strength slightly but lowers the workability.

Fig. 14 shows the evaluated void ratio from the experiments using mixtures of both natural sand (0-2 mm) and crushed rock (Käll. 0-2 mm). The blue curve represents the presently proposed model for this mixture. It gave a minimum void ratio of when the natural sand ratio is 45% in the mixture.

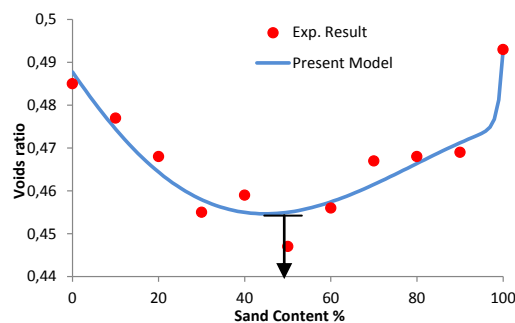


Fig.14. Effect of sand material on the experimentally estimated void ratio mixed with Käll. 0-2 mm crushed material.

The concrete mix recipe used in these tests was as follow: (18.4 kg aggregate), (3.2 kg Portland cement type CEM 11/ A-LL 42.4 R Skövde) and (2.1 l water).

The void ratio, fineness modulus, slump and the remolding number had been evaluated and tested for a number of mixes based on the aggregates and aggregate proportions shown in Tables 2 and 3. Table 4 and Fig. 15 show the evaluated test results (Kristoffer, 2005).

Table 2: Aggregate properties aimed to use in concrete properties tests

Aggregate type	Aggregate size (mm)	Symbol	Void ratio	Fineness modulus
Kåll. Crushed, (Fig. 5)	0-2	A	0.447	2.808
Sand, (Fig. 5)	0-2	B	0.493	1.533
Kåll. Crushed, (Fig. 6)	2-5	C	0.475	4.978
Kåll. Crushed, (Fig. 6)	4-8	D	0.488	5.484
Vikan crushed, (Fig. 6)	11-16	E	0.467	7.503
Modified, Aggregate Mix 1	70% A+ 30% B	F	0.436	2.425
Modified, Aggregate Mix 2	50% A + 50% B	G	0.419	2.170
Modified, Aggregate Mix 3	60% C+ 40% D	H	0.456	5.180

Table 3: Aggregate mixture proportions used in concrete mix

Concrete mix 1		Concrete mix 2		Concrete mix 3		Concrete mix 4		Concrete mix 5		Concrete mix 6	
Mix ratio (%)	Agg. type	Mix ratio (%)	Agg. type	Mix ratio (%)	Agg. type	Mix ratio (%)	Agg. type	Mix ratio (%)	Agg. type	Mix ratio (%)	Agg. type
40	A	50	A	28	A	20	A	28	A	20	A
20	H	10	H	12	B	20	B	12	F	20	G
40	E	40	E	20	H	20	H	20	H	20	H
				40	E	40	E	40	E	40	E

Table 4: Test results for concrete properties

Test types	Concrete mix 1	Concrete mix 2	Concrete mix 3	Concrete mix 4	Concrete mix 5	Concrete mix 6
Aggregate void ratio	0.327	0.323	0.334	0.319	0.340	0.338
Fineness modulus	5.160	4.923	5.007	4.905	5.114	5.032
Remolding number	21	44	21	12	29	28
Slump (mm)	40	30	140	170	50	65
Compressive strength (MPa)	33.9	34.9	31.7	30.0	32.9	33.3

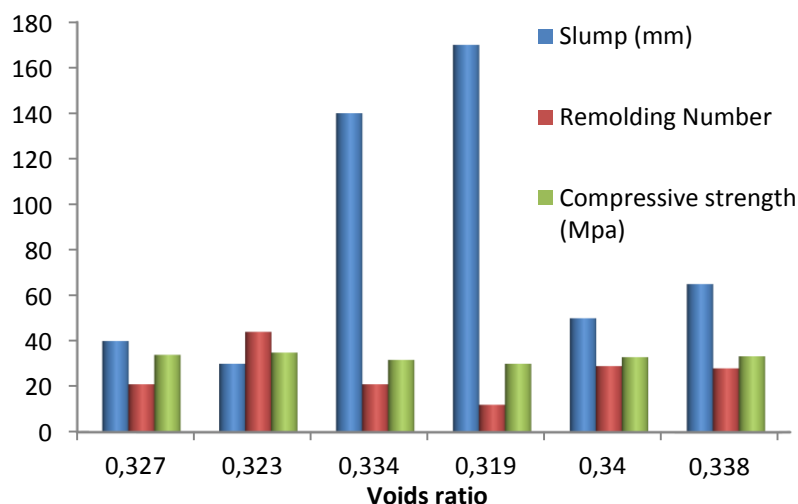


Fig.15. Concrete properties results based on void ratios.

From Table 4 and Fig. 15 it can be noticed that the addition of sand (0-2 mm) had a positive effect on mixes 3, 4 (140, 170 mm slump) compared with the original mixes 1 and 2 (40, 30 mm slump). This illustrates that rounded fine aggregate grains gave positive effect

compared to the crushed material with the same grain size. This is due to the fact that natural sand has greater ability to move into voids more easily than that with edged grains.

Similarly, it was found that the lower remolding number for the rounded sand gives higher workability.

For mixes containing rounded sand the average compressive strength was lower than that of the others, which can be explained by smaller impact of the intergranular friction (Kristoffer, 2005).

6.4.2 Proposed theoretical model of compressive strength

To realize the overall effect and individual effects of independent variables (void ratio, remolding number, slump and fineness modulus) on the compressive strength, the analysis via Anova of the statistical software Minitab 16 was applied to find out which factors or interaction effects on the concrete strength are most important. It was found that with 95% confidence intervals, the slump has no significant effect on the compressive strength and it was hence not included in the model. Regression analysis was applied using the same program in order to find the best theoretical model for the compressive strength and the model defined as Eq. 5. The one having a correlation coefficient of 0.94 was found to be superior.

$$f_c' = -2.1 - 63.8 V + 0.150 R + 10.4 FM \quad (5)$$

where f_c' : Compressive strength, cube sample, MPa; V : Void ratio; R : Remolding number; FM : Fineness modulus.

6.4.3 Cement paste content based on void ratio

The paste, cement and water work together as lubricate in the fresh concrete. It is important to know the effect of the amount of paste, which is usually decided by experience. The common principle is to balance the amounts of constituents so as to have the voids in the aggregate mixture filled by paste. The present work started by selecting a void ratio of 0.35 (35 %) requiring an amount of paste of 65 % by volume to fill the voids. Applying these ratios to a system having 20 l volume the aggregate will represent 13 l and the voids 7 l, which will be filled with cement paste (Kristoffer, 2005).

The aggregate used in this study was of type 3 Enhörna, cf. Fig. 3 and the recipe was as shown in Table 5. The fractions 0-4 mm and 4-8 mm were not constant, but the coarsest fraction 8-11 mm was kept constant at 40 % as specified in the table. The concrete mix volume, consisting of paste and aggregate was 20 l. For checking the accuracy of the modeling three paste volumes were assumed: one original paste volume and the others were changed by increasing and decreasing the paste volume by 0.8 l, thereby creating three systems. The first, second and third were filled 96%, 100 % and 104% of the paste volume respectively (Table 6) (Kristoffer, 2005).

Table 5: Aggregate properties

Concrete Mix	A1	A2	A3	A4
Void ratio	0.354	0.364	0.387	0.422
Aggregate proportions %	60-0-40	50-10-40	40-20-40	30-30-40
Fineness modulus	3.962	4.436	4.588	4.901

Table 6: Concrete mix components and proportions (kg)

Concrete mix	(-) 0.8L (volume of cement paste)			Original volume			(+) 0.8L (volume of cement paste)		
	Cement	Water	Agg.	Cement	Water	Agg.	Cement	Water	Agg.
A1				7.33	4.69	34.24	8.16	5.22	34.24
A2	6.74	4.29	33.71	7.57	4.82	33.71	8.40	5.35	33.71
A3	7.18	4.60	32.49	8.01	5.13	32.49	8.84	5.66	32.49
A4	7.91	5.06	30.63	8.74	5.59	30.63			

Slump test had been made to find out the function of the cement paste in the preparation phase. This required two tests, one after 3 min and the other after 8 min of blending using 20 l of concrete for each of them. Variance was found and is illustrated by Table 7. This is due to the difference in chemical reactions of the cement and the loss of fluid by leakage from the mixer. A second reason is that measuring slump after 8 minutes did not give exact values because the fluid consisted of only water and cement. Fig. 16 shows the slump results plotted against void ratio (Kristoffer, 2005).

Table 7: Slump test results (mm)

Concrete mix	Slump, (-) 0.8L (volume of cement paste)		Slump Original volume		Slump, (+) 0.8L (volume of cement paste)	
	3 minute	8 minute	3 minute	8 minute	3 minute	8 minute
A1			90	60	180	140
A2	70	45	160	90	205	170
A3	185	125	205	140	225	220
A4	215	165	240	195		

The results presented in Table 7 and Fig. 16 implies high fluidity. This is due to the high values of slump results for pilot mixes (A3 and A4). This means strong separation of the concrete components. Oversaturation of the aggregate voids with cement paste naturally gives a high void ratio (Kristoffer, 2005).

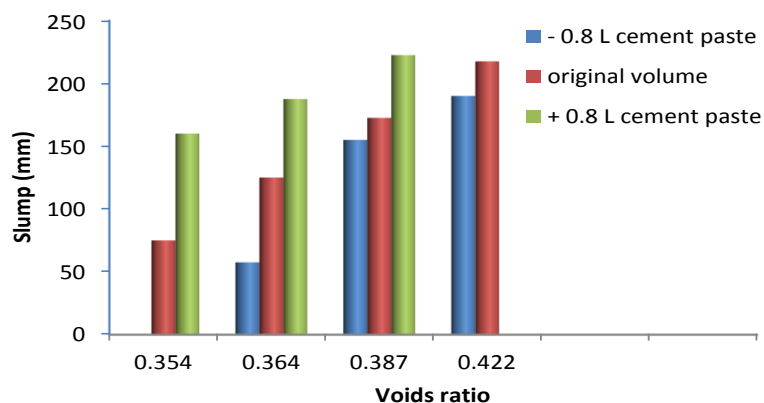


Fig.16. Slump test results versus voids ratio for different amounts of paste.

6.4.4 Proposed theoretical models of cement content and slump

Theoretical models were constructed (based on experimental work lead to having Eq. 6 and 7) using Minitab 16 for statistical treatment and regression analysis. The first model using Eq. 6, refers to the slump value while the second using Eq. 7, gives the cement content. The correlation factors were $R^2 = 91\%$ and 100% , respectively.

$$S = -488 - 717 V + 106 FM - 8.5 T - 674 C + 1150 W \quad (6)$$

$$C = 0.0431 - 1.32 V - 0.000125 S + 0.0952 FM + 1.57 W \quad (7)$$

Where: S : Slump (mm); V : Void ratio; FM : Fineness modulus; T : Time (min.); C : Cement content (kg); W : Water content (kg).

7. Conclusions

A number of user-friendly software codes and mathematical models are available on the market, offering tools for finding constituent proportions of concrete with a minimum void ratio. It is obvious that the natural aggregate gives lower void ratios than the crushed aggregate for the same aggregate mix proportions for all cases. It is concluded that the fine aggregate content controls most of the properties of the concrete, and that a suitable content of fine aggregate ranges between 40% and 60% of the total aggregate content for obtaining a low void ratio of the aggregate component. The results showed the importance of having sufficient amounts of filler material and cement paste for separate larger particles and for effective mutual binding. Hence, aggregate voids should be slightly “overfilled” for giving the concrete good workability and fluidity at casting. It is also found that the use of sand as substitute for crushed material improves very much the workability of concrete and helps to give a low void ratio. The reason is easy slip of smooth, rounded grains. Finally, slump test show that the concrete with optimal aggregate distribution, minimum voids and sufficient content of fine particles needs only a small amount of cement paste to give it high strength.

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