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Analysis of a Coriolis Acceleration Ibrahim

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ABSTRACT

The velocity of a moving point in a general path is its vector quantity, which has both magnitude and direction. The velocity vector can change over time because of acceleration, which can be tangential, radial and Coriolis types. Acceleration analysis is important because inertial forces are proportional to their rectilinear, angular and Coriolis accelerations. The loads must be determined in advance to ensure that a machine is designed to handle these dynamic loads. For planar motion, the vector direction of acceleration is separated into the tangential, radial and Coriolis components of a point on a rotating body. The Coriolis acceleration is the product of linear and rotational velocities. All textbooks in physics, kinematics and machine dynamics consider the magnitude of a Coriolis acceleration at a common condition when an object moves and rotates at variable velocities. The magnitude of the Coriolis acceleration is considered on a basis of that common condition. However, the magnitudes of Coriolis acceleration vary according to the conditions of motions of an object. This paper presents new analytical expressions of the Coriolis acceleration under the conditions of uniform, accelerated and combined motions of an object and a rotating element and thereby fills a gap in the study of acceleration analysis.

Keywords: Coriolis acceleration, force, kinematics and dynamics of mechanisms.

1. Introduction

Machines and mechanisms running at a high speed are considered by a kinematic analysis of their velocities and accelerations, and subsequently analyzed as dynamic systems in which forces due to accelerations are analyzed using the principles of kinetics. Force and stress analysis is based on the acceleration analysis of links and points of interest in the mechanism or machine.

In classical mechanics there are many ways to derive equations of accelerations. The Coriolis acceleration is the product of the linear motion of an object on the rotating table

(Gregory, (2006); Hibbeler, (2005); Myszka, (2005); Norton, (2002); Syngley and Uicker, year; Soutas-Little et al., (2008); Taylor, (2005); Wilson and Sadler, (2003); Waldron and Kinzel 2004) . Textbooks in classical mechanics calculate the Coriolis acceleration and a force for common case as follows:

$$a_c = 2V\omega$$
$$F_c = 2mV\omega \qquad (1)$$

where a_c is the Coriolis acceleration, ω is the angular velocity of the disc, and V is the linear velocity of the object travelling on the table, m is the mass of the object, and F_c is the Coriolis force.

Equation 1 is a classical formula. The components of the angular velocity ω and the linear velocity V are applied for both uniform and accelerated motions of machine elements. However, this approach is not logically correct, because any differences in motions of elements of mechanisms should be reflected in the equations that describe these motions. Following analysis of examples of the Coriolis accelerations show shows that for different cases of motions the magnitude of this acceleration is different. For example, mechanisms, which have Coriolis effects, can have the linear and rotating motions that are uniform and/or accelerated.

Some textbooks derive Coriolis acceleration and its force on a base of the uniform velocities. An object of the mass m travels from the centre ***O*** towards B on the edge of a rotating disk as shown in Figure 1.

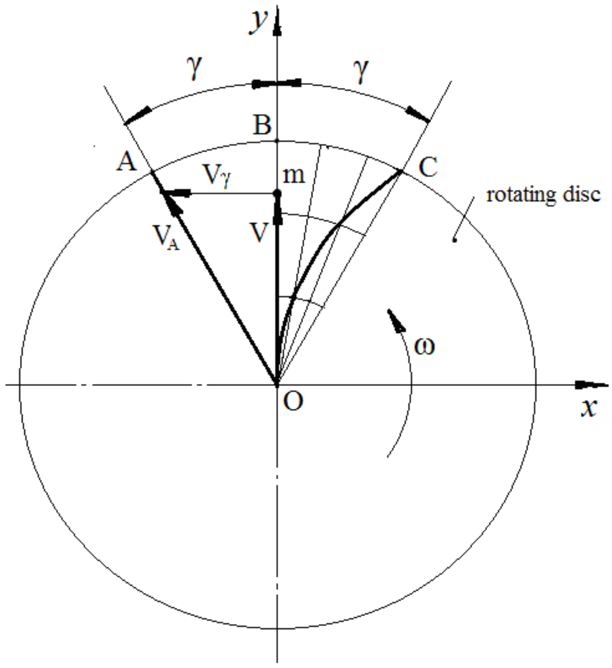

Figure 1. Sketch for calculation of the Coriolis acceleration

The linear velocity V of the object m and angular velocity ω of the disc is uniform. The travelling from point O to B takes time Δt, so the distance OB = V*Δt . The distance OB is calculated by calculated by the physics equations of the uniform motion, OB = V*Δt. The rotation of the table results, the point B will have moved from its original position to a new position C after a time Δt. In angle $\gamma = \omega\Delta t$, ω is the uniform angular velocity. However, the distance BC is calculated by the physics equations of the accelerated motion, BC = αt2/2. Finally, the Coriolis acceleration for this type of approach is depicted in Eq. (1).

However, the following analysis of the accepted motions of the object *m* on a rotating disc shows the following circumstances (Fig. 1):

a) The motion of the object *m* and the rotation of the disc are independent and no forces are acting between the object *m* and the disc.
b) The linear motion of the object *m* and the rotary motion of the disc are uniform, hence the relative trajectory of a motion *OC* of the object *m* on the rotating disc is the Archimedes curve. The principle of the Archimedes curve states that any point of the curve gives uniform velocities of components and without acceleration.
c) The distance *BC* is a result of uniform motions and calculated by the equation $BC = \omega*(OB)*\Delta t = V_{tB}*\Delta t$, where V_{tB} is the constant tangential velocity of the point *B.*
d) The distance *BC* is the length between the point *B* of disc and the point *C* of the Archimedes curve as calculated by Eq. (2), which expresses the rotation of the disc with acceleration that contradict to the accepted initial conditions.
e) These circumstances show that Eq. (2), which describes the motion with acceleration cannot be used for analysis and for the following derivation of the Coriolis acceleration.

Analysis of the physics of two motions and the Coriolis acceleration and its force for represented example is not so strong mathematically. Free linear motion with uniform velocity of the object *m* on the rotating disc of uniform angular velocity does not create accelerations and therefore force. The Coriolis acceleration and its force act when the object *m* travels on the guide way of the rotating disc. The linear velocity of the object and angular velocity of the rotating disc can be uniform and/or accelerated.

The object, which moves on the rotating table, can test different magnitudes of Coriolis acceleration and a force. This study examines three types of motions:

a) uniform motions of the object and the table
b) uniform motion of the object and the accelerated rotation of the table
c) accelerated motion of the object and the uniform rotation of the table

2. Analytical approach

2.1. Coriolis effect for uniform motions of the object and the rotating table.

The object *m* travels on the rotating disc and its trajectory represented by the line *OA* (Fig. 1). Rotation of the disc on the angle γ changes the vector velocity direction *V* of the object *m* on the vector velocity direction V_A. This change in the direction of the vector velocity is represented by the vector V_γ, whose expression is represented by the following equation:

$$V_\gamma = -V\sin\gamma \quad (2)$$

where γ is an angle of rotation of a disc, other components are as specified above.

The magnitude of the vector velocity V_γ depends on the change in the object's radius location on the disc. The change of magnitude of the velocity V_γ is represented by the following equation [10]:

$$\Delta V_\gamma = -[V\sin(\gamma + \Delta\gamma) - V\sin\gamma] = -V[\sin(\gamma + \Delta\gamma) - \sin\gamma] \tag{3}$$

Trigonometric expression in square brackets is represented by the identity $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$ [10]. After substituting this identity into Eq. (3), and multiplying by $\left(\frac{\Delta\gamma/2}{\Delta\gamma/2}\right)$ the change of the tangential velocity V_γ has the following equation:

$$\Delta V_\gamma = -V*2\cos\left(\frac{\gamma+\Delta\gamma+\gamma}{2}\right)\sin\left(\frac{\gamma+\Delta\gamma-\gamma}{2}\right)\left(\frac{\Delta\gamma/2}{\Delta\gamma/2}\right)$$

$$\Delta V_\gamma = -V*2\cos\left(\gamma+\frac{\Delta\gamma}{2}\right)\sin\left(\frac{\Delta\gamma}{2}\right)\left(\frac{\Delta\gamma/2}{\Delta\gamma/2}\right) \tag{4}$$

Analysis of Eq. (6) shows, when $\Delta\gamma \to 0$, then $\lim_{\Delta\gamma\to 0}\cos\left(\gamma+\frac{\Delta\gamma}{2}\right) = \cos\gamma$, $\lim_{\Delta\gamma\to 0}\frac{\sin(\Delta\gamma/2)}{\Delta\gamma/2} = 1.0$. Substituting the defined expression into Eq. (6) and then transforming

$$\Delta V_\gamma = -V*2\cos\gamma*(\Delta\gamma/2) = -V\cos\gamma*\Delta\gamma \tag{5}$$

Analysis assumes the angle γ is a small value, hence, it is accepted that $\cos\gamma = 1$. Hence, the differential equation of the change of the velocity will have the following equation:

$$dV_\gamma = -Vd\gamma \tag{6}$$

The rate change of the velocity per time represents an acceleration of the object, which expression has the following equation:

$$\frac{dV_\gamma}{dt} = -V\frac{d\gamma}{dt} \tag{7}$$

where $dV_\gamma/dt = a_c$ is the acceleration of the object, $d\gamma/dt = \omega$ is the angular velocity of the rotating disc.

Substituting defined components into Eq. (7) and transforming the acceleration of the object results in the following expression:

$$a_c = -V\omega \tag{8}$$

Eq. (8) is expression of the Coriolis acceleration for uniform motions of the object and the table. Hence, the Coriolis force will have the next equation

$$F_c = mV\omega \tag{9}$$

The direction of the Coriolis force vector is opposite to the direction of the vector of acceleration.

Eq. (8) can be derived by another mathematical approach. The distances *AB* and *OB* can be calculated by the equations of AB = $V_\gamma t$, and $OB = Vt$. The angle γ of triangle *AOB* is small, then $AB = OB\sin\gamma$, and accepted $\sin\gamma = \gamma$. After substituting defined magnitudes of the triangle sides, the new equation has the following expression: $V_\gamma t = -Vt\gamma$.

After transformation of this expression, the rate change of the velocity per time represents an acceleration of the object, which expression is the same as Eq. (11) and Coriolis acceleration expressed by Eq. (8).

2.2. Coriolis effect for the uniform motion of the object and the accelerated rotation of the table

Rotation of the disc is variable and an angular acceleration changes the magnitude of the vector velocity $V_\gamma = a_\gamma t$ of the object *m* (Fig. 1). The distance *AB* can be calculated by the equation of a variable motion $AB = a_c t^2/2$, and distance *OB* by the equation of a uniform motion $OB = Vt$. The angle γ of triangle *AOB* is small, then $AB = OB\sin\gamma$, and accepted $\sin\gamma = \gamma$. After substituting defined magnitudes of the triangle sides

$$\frac{a_c t^2}{2} = -Vt\gamma \tag{10}$$

where $\gamma = \varepsilon t^2/2$ is an angle of rotation of a disc with acceleration ε, other components are as specified above.

After substitution and the following transformationgive equation of the Coriolis acceleration and its force for the uniform motion of the object and the accelerated rotation of the table

$$a_c = -Vt\varepsilon \tag{11}$$
$$F_c = mVt\varepsilon$$

2.3. Coriolis effect for the accelerated motion of the object and the uniform rotation of the table

Rotation of the disc is uniform and a vector of the object is variable. The equation of the vector velocity $V_\gamma = a_\gamma t$ of the object m (Fig. 1). The distance AB can be calculated by the equation of a variable motion $AB = a_c t^2/2$, and distance OB by the equation of an accelerated motion $OB = at^2/2$. If angle γ of triangle AOB is small, then $AB = OB\sin\gamma$, and accepted $\sin\gamma = \gamma$. After substituting defined magnitudes of the triangle sides,

$$\frac{a_c t^2}{2} = -\frac{at^2}{2}\gamma \tag{12}$$

where $\gamma = \omega t$ is a variable angle of uniform rotation of a disc, other component are as specified above, a is an acceleration of the object; other components are specified above.

The following transformation gives equation of the Coriolis acceleration and its force for the uniform motion of the object and the accelerated rotation of the table

$$a_c = -a\omega t \tag{13}$$
$$F_c = ma\omega t$$

2.4. Coriolis effect for the accelerated motions of the object and rotating table

Rotation of the disc is accelerated and a vector of the object is variable. The magnitude of the vector velocity $V_\gamma = a_\gamma t$ of the object m (Fig. 1). The distance AB can be calculated by the equation of a variable motion $AB = a_c t^2/2$, and distance OB by the equation of an accelerated motion $OB = at^2/2$. The angle γ of triangle AOB is small, then $AB = OB\sin\gamma$, and accepted $\sin\gamma = \gamma$. After substituting defined magnitudes of the triangle sides

$$\frac{a_c t^2}{2} = -\frac{at^2}{2}\gamma \tag{14}$$

where $\gamma = \varepsilon t^2/2$ is the angle of rotation of the table with acceleration, other components are as stated.

After substituting and the following transformation give the equation of the Coriolis acceleration and its force for the uniform motion of the object and the accelerated rotation of the table

$$a_c = -a\varepsilon t^2/2 \tag{15}$$
$$F_c = ma\varepsilon t^2/2$$

Table 1: Coriolis Acceleration for Uniform and Accelerated Motions of the Object and the Rotating Table

Motion	Uniform		Uniform	Accelerated	Accelerated	Uniform	Accelerated	
	Object	Table	Object	Table	Object	Table	Object	Table
Coriolis acceler-n	$a_c = -V\omega$		$a_c = -Vt\varepsilon$		$a_c = -a\omega t$		$a_c = -a\varepsilon t^2/2$	
V, a	Linear velocity and linear acceleration of the object accordingly							
ω, ε	Angular velocity and angular acceleration of the rotating table accordingly							
T	time							

3. Result and discussion

Relevant equations produced for Coriolis accelerations of a moving object on a rotating table enable calculation of more accurate results. New equations consider uniform and accelerated linear and the angular velocities of a mechanism. These equations are different from the fundamental equation of Coriolis acceleration depicted in textbooks of physics, kinematics and machine dynamics. In engineering, all machines work with variable velocities of elements that is real condition of functioning of any mechanism. It is very important to calculate an exact result of acting forces for machine components with acceleration. The new equations of Coriolis accelerations give a more accurate result than do those presented in the textbooks Gregory, (2006); Hibbeler, (2005); Myszka, (2005); Norton, (2002); Syngley and Uicker, year; Soutas-Little et al., (2008); Taylor, (2005). Example of the results of calculation of the Coriolis accelerations represented in the Table 1.

The rotating table shown in Fig.1 has the angular velocity of 2.0 rad/s, accelerates at the rate of 3.0 rad/s^2. The object with of 0.1 kg moves with linear velocity 0.5 m/s, accelerates at the rate of 1.0 m/s^2. Determine the magnitudes of the Coriolis accelerations of four types of motions after 3.0 s of motions (Table 1).

The results of calculations by the new Equations (8), (11), (13) and (15) obtained for Coriolis acceleration of an object, which moves on the rotating table and calculated by Equation (1) of textbooks are different as shown in Table 2. These are differences in approach of analysis of Coriolis. The new analysis of Coriolis acceleration considers different conditions of the work of mechanism.

Table 2: Results of the Coriolis Acceleration

Motion	Uniform		Uniform	Accelerated	Accelerated	Uniform	Accelerated	
	Object	Table	Object	Table	Object	Table	Object	Table
Coriolis Acceleration	$a_c = -V\omega =$ $-0.5 m/s * 2 rad/s =$ = -1.0 m/s^2		$a_c = -Vt\varepsilon =$ -0.5m/s*3s*3rad/s^2 = -4.5 m/s^2		$a_c = -a\omega t =$ 1.0m/s^2*2.0rad/s*3.0s= -6 m/s^2		$a_c = -a\varepsilon t^2/2 =$ 1.0m/s^2*3rad/s^2 *(3s)^2/2= -13.5 m/s^2	
	Equation of textbooks $a_c = -2V\omega$ = -2*0.5m/s*2.0rad/s = -2.0 m/s^2							

4. Conclusion

Fundamental analytical solutions of analysis of Coriolis acceleration that are presented in textbooks cannot give accurate equations for variable conditions of the machine work. The main contribution of this paper is the derivation of new equations of Coriolis acceleration for different conditions of the mechanism work. The paper presents an original analysis of Coriolis acceleration and new analytical equations of the accelerations giving accurate calculations.

The new analysis of Coriolis acceleration should be used in textbooks of physics, kinematics and machine dynamics. The new analytical approach gives accurate mathematical method for analysis of Coriolis acceleration and correct results in calculation. These new equations of Coriolis accelerations can be used by engineers and manufacturers for production of machines that will have better technical and economic characteristics.

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