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Qotboldin Shirazi and Euclid's Elements

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ABSTRACT

The Book of Euclid's Elements, named after its author and also called "The Book of Elements" in the mathematical publications, is one of the important books of the Islamic period. It was translated several times and those translations were also modified several times and many important and useful comments were added on it by some scholars. The present paper aims to analyze a united shape resulting from the integration of the forty eight fold shapes of the theorem of Euclid's elements. This shape was drawn by Qotboldin Shirazi, one of the translators of the Euclid's Elements from Arabic into Persian.

***Keywords*: Euclid's Elements; Qotboldin Shirazi; Translation; Elements; The Book of Elements; Mathematics of Islamic Era**

1. Introduction

The Muslims' access to the Greek sciences including mathematics was started in particular from the Abbasid era. Mansour (136-158 A.H./754-775 A.D.) and Al-Mamoun, (198-218 A.H./813-833 A.D.) of the Abbasid Caliphates got some copies of Euclid's Elements, some rules written about the geometry by Euclid[1] in Byzantium along with other copies of other Greek works (Ghorbani, 1987).

The Euclid's Elements was translated at the age of caliphate of Haroun al Rashid 170-192 A.H./786-809 A.D) by Hajaj ibn Yusef ibn Matar and was named as "Harouni". He also compiled an abridged version of the Elements with modifications and explanations for Mamoun which is called "Mamouni".

The other translation is a work by Ishaq ibn Honain ibn Ishaq al Ebadi (d/298 A.H/910 A.D.). This translation which was revised by Sabit ibn Qorah seems to be a sample of a good

[1]Euclid, the Greek mathematician who possibly used to live at the age of Ptolemy, the first (283-306 B.C.) in Alexandria. He compiled the knowledge of his age in the Book of Euclid's Elements which includes thirteen articles. This book has been the source of education of preliminary geometry from long time ago up to the present and a great part of it is the work of Euclid himself. It is a book including the most advanced degree of mathematical sciences of that age in which Euclid has shown a great talent in preparing them.

translation. While trying to remove the barriers of translation in the Greek text, translator has rendered the contents in the Greek language into Arabic faithfully.

The third translation of existing Arabic translation is by Khajeh Nasiruldin Toosi (b. 597 A.H/1201 A.D). That is not a translation but in fact a kind of authorship and rewriting based on older Arabic translations. That translation is usually called as Nasir al-Din al-Tusi's recension of Euclid's Elements (Ghorbani, 1987).

The above translation from Euclid's Elements became bases for other translations. On this basis, the first Latin translation which is a complete translation was not made from Greek but rather from the Arabic language (Heath, 2002).

Among other important translations from Arabic translations are two Persian translations which were made by Qotboldin Shirazi[2] (634-710 A.H) (Karamati, 2005).

The first translation is the Persian Nasir al-Din al-Tusi's recension of Euclid's Elements which was made in 681 A.H. (Ghorbani, 1968)

The second translation is the Euclid's Elements by Qotboldin Shirazi which was included in his encyclopedia "Doratoltaj"[3] Though at a glance, it seems the second translation is in fact the very same translation [Ghorbani,1987; Ghorbani, 1968; Meshkat, 1978-1981; Fabio, 2008]with a very few ignorable differences. They have been included in the part on mathematics at Dorat al Taj due to the importance of Euclid's Elements.

Other noticeable feature in both translations by Qotboldin is the application of certain innovations including the integration of shapes of different theorems.

2. Compact Diskette [Abridged Table] of 48 Theorems

Qotboldin Shirazi employed the above method of integrating the shapes for the 48 geometrical theorems of the first article of Euclid's Elements[4].

It is worth mentioning that the Euclid's Elements includes thirteen articles and each article relatively includes the theorems related to a geometrical domain.

The theorems of the first article are placed in three distinctive groups. First are the theorems which are mainly related to triangles, drawing and their features, i.e the relationship existing among the components, i.e sides and their angles with each other along with three theorems related to the ability of adaptation of shapes with each other. There is also discussion on the meeting of two straight lines with each other and reciprocating angles to the top and adjunct angles which are made by two lines. It also includes a few numbers of simple drawing issues, and drawing a line perpendicular on other line and drawing a bisector of an angle and a perpendicular bisector of a straight line.

[2] Mahmoud ibn Masoud ibn Mosleh known as Qotoboldin Shirazi (634-710 A.D.), Iranian physician, mathematician, astronomy scientist, physic scholar and philosopher. He was one of the students of Khaje Nasiruldin Toosi.

[3] This book whose date of compilation is between 693 to 705 A.H. was also known as: Alnamoozaj al Oloum for having all types of theoretical and practical wisdom.

[4] The author has referred to the first article of six copies of Euclid's Elements of Khajeh Nasir in order to study the authenticity of attribution of it to Qotboldin Shirazi which has been added to translation. The six studied copies all belong to the Library of Astan-e Qods-e Razavi under numbers 5443, 5444, 5445, 5446, 5261 and 7483.

The second group established the theories of parallels. The third group introduces for the first time the parallelograms and discusses in general the parallelograms, triangles, squares and their surface area.

The first article which was precisely referred to in the three fold groping has 48 theorems which include[5]:

The first theorem: We want to make an equilateral triangle on an assumed line segment.

The second theorem: We want to draw from an assumed point a line segment equal to another assumed line segment.

The third theorem: Two line segments are assumed such than one of them is lager than the other one. We want to divide on the larger line segment equal to the smaller line segment.

The fourth theorem: Whenever two sides and the angle between from a triangle is equal with two sides and between angle of another triangle, then the sides and other angles and also two triangles are equal.

The fifth theorem: In an isosceles triangle, the neighboring angles with the base are equal and also the two angles below base which had been emerged from the continuation of the two sides are equal with each other.

The sixth theorem: If two angles of a triangle are equal with each other, the opposite sides to those angles are also equal.

The seventh theorem: A line segment is assumed. We have drawn two lines from two ends of assumed line segments in one side of the side segment which have cut each other in a point. At the same side, once again, we have drawn from two ends of the same line segment, two other lines which cut each other in another point. We want to prove that the drawn line segments from the one end of the assumed line segment are not equal with each other.

The eighth theorem: Whenever the sides of a triangle are not equal with the sides of another triangle, then the peer angles and also two triangles will be equal with each other.

The ninth theorem: We want to draw a bisector of the assumed angle.

The tenth theorem: We want to half an assumed line segment.

The eleventh theorem: We want to draw a perpendicular from an assumed point on a line.

The twelfth theorem: We want to make a perpendicular from a point not located on a line.

The thirteenth theorem: Whenever a line is perpendicular on another line, the two created angles are either perpendicular or their sum is equal to two perpendiculars.

The fourteenth theorem: If we draw two lines in both sides of an assumed line from the assumed point located on an assumed line, and the sum of the created two angles is equal to two perpendiculars (rights), then the two drawn lines from the assumed point of a line are straight.

The fifteenth theorem: Two angles opposite to the top are equal.

The sixteenth theorem: The created external angle from the continuation of one of the sides of triangle is bigger than any non-neighboring internal angle of that triangle.

The seventeenth theorem: In each triangle, the sum of each of both angles is less than two perpendiculars.

[5] Author has rewritten the following theorems based on the copy of No. 11985 of Astan-e Qods-e Razavi in modern language.

The eighteenth theorem: In each triangle, the angle opposite to the larger side is bigger than the angle opposite to the smaller side.

The nineteenth theorem: In each triangle, the side opposite to the bigger angle is bigger than the side opposite to the smaller angle.

The twentieth theorem: In each triangle, the sum of each of both sides is bigger than the third side.

The twenty first theorem: Whenever we draw two lines from two ends of a side of triangle to cut each other inside the triangle, the sum of the two created line segments will be smaller than the sum of the two other sides of the triangle but the angle between those two will be bigger than the angle of the third triangle.

The twenty second theorem: We want to draw a triangle whose sides to be equal to three assumed line segments in which the sum of the both line segments to be bigger than the third line segment.

The twenty third theorem: We want to draw an angle equal to an assumed angle from a point located on an assumed line.

The twenty fourth theorem: Whenever two sides of a triangle to be equal with two sides of another triangle, but the angle between them to be unequal, the side opposite to the bigger angle is bigger than the side opposite to the smaller angle.

The twenty fifth theorem: Whenever two sides of a triangle to be equal with two side of another triangle, but their third sides not to be equal, then the angle opposite to the bigger side is bigger than the angle opposite to the smaller side.

The twenty sixth theorem: Whenever two angles and one side of a triangle to be equal with two angles and one side of another triangle, then the other angles and sides and consequently the two triangles will be equal with each other.

The twenty seventh theorem: Whenever two lines cut a line and two equal internal corresponding angles to be created, those two lines will be parallel.

The twenty eight theorem: Whenever two lines cut a line and two internal and external opposite angles to be equal, or the two internal opposite angles to be two right angles, then those two lines will be parallel with each other.

The twenty ninth theorem: If two parallel lines cut a line, the two internal corresponding angles and two internal and external opposite angles will be equal with each other and the two internal opposite angles will be equal to two right angles.

The thirtieth theorem: The lines parallel with a line are parallel with each other.

The thirty first theorem: We want to draw an assumed line from an assumed line of a parallel line.

The thirty second theorem: In each triangle, the external angle is equal to the sum of the two non-neighboring internal angles and the sum of the internal angles of the triangle is two perpendiculars.

The thirty third theorem: Two line segments which connects the ends of two parallel and equal line segments with each other are parallel and equal themselves.

The thirty fourth theorem: In a parallelogram, the opposite sides and their opposite angles are equal and the diameter halves the parallelogram.

The thirty fifth theorem: The parallelograms which have a common base and the sides parallel with base in them are located on a line, they will be equal.

The thirty sixth theorem: The parallelograms whose bases are equal and located on a line and the sides parallel with base in them are also located on a line, they will be equal.

The thirty seventh theorem: The triangles which have a based and the opposite tops to based in them are located on a straight line parallel with based will be of the same area with each other.

The thirty eight theorem: The triangles shoes bases are equal and located on a line and their opposite tops to based on them are located on a line parallel with based are of the same area.

The thirty ninth theorem: The trainable with the same area which have one based and located in one side of bases, their tops will be also located on a line parallel with the base.

The fortieth theorem: The triangles with the same areas whose bases are equal and located on one line and in one side of that line, their tops also are located on a line parallel with the same line.

The forty first theorem: If a parallelogram and a triangle has one base and the top of the triangle to be located on the continuation of the side parallel with the base from the parallelogram, then the parallelogram will have an area two times of the triangle areas.

The forty second theorem: We want to draw a parallelogram to be with equal surface areas of that of the assumed triangle and one of its angles to be equal with the assumed angle.

The forty third theorem: In each parallelogram, the supplements of parallelogram round a diameter are of the same surface area.

The forty fourth theorem: We want to draw a parallelogram with the same area of an assumed triangle on an assumed line, such that one of its angles to be equal with the assumed angle.

The forty fifth theorem: We want to draw a parallelogram with the same surface area of an assumed parallelogram with an angle equal with an assumed angle

The forty sixth theorem: We want to draw square on an assumed line segment

The forty seventh theorem: In each right angled triangle, the square of the side opposite to the right angle (hypotenuse) is equal with the sum of the squares of the two other sides.

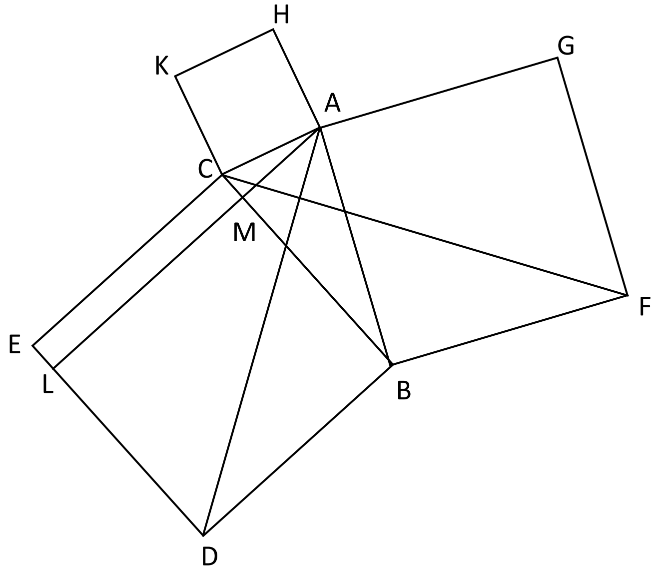
The forty eight theorem: If in a triangle, the square of a side is equal to the sum of the squares of the two other sides, then that triangle is right angled one.

In order to prove the above mentioned 48 theorems, there is a need to draw the proper shapes of each theorem and Qoboldin Shirazi has integrated the essential shapes of 48 theorems with each other and thereby has presented the following shapes.

Qutb al-Din Shirazi has displayed 48 shapes relating to 48 geometrical theorems of the first article of Euclid’s Elements in a unit shape that in this paper we have named it “Compact Diskette (disk-disc)”.

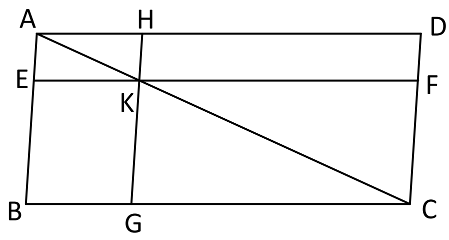
For example we prove 43rd and 47th theorems, which follow.

43. In each parallelogram, the supplements of parallelogram round a diameter are of the same surface area.



Let *ABCD* be a parallelogram and *AC* its diameter; and about *AC* let *EH*, *FG* be parallelogram, and *BK*, *KD* the so-called complements; I say that the complement *BK* is equal to the complement *KD*.

For, since *ABCD* is a parallelogram, and *AC* its diameter, the triangle *ABC* is equal to the triangle *ACD*. Again, since *EH* is a parallelogram, and *AK* is its diameter, the triangle *AEK* is equal to the triangle *AHK*.



For the same reason the triangle *KFC* is also equal to *KGC*.

Now, since the triangle *AEK* is equal to the triangle *AHK*, and *KFC* to *KGC*, the triangle *AEK* together with *KGC* is equal to the triangle *AHK* together with *KFC*. And the whole triangle *ABC* is also equal to the whole *ADC*; therefore the complement *BK* which remains is equal to the complement *KD* which remains.

Therefore etc.

47. In each right angled triangle, the square of the side opposite to the right angle (hypotenuse) is equal with the sum of the squares of the two other sides.

Let *ABC* be a right-angled triangle having the angle *BAC* right; I say that the square on *BC* is equal to the squares on *BA*, *AC*.

For let there be described on *BC* the square *BDEC*, and on *BA*, *AC* the squares *GB*, *HC*; through *A* let *AL* be drawn parallel to either *BD* or *CE*, and let *AD*, *FC* be joined.

Then, since each of the angles *BAC*, *BAG* is right, its follows that with a straight line *BA*, and at the point *A* on it, the two straight lines *AC*, *AG* not lying on the same side make the adjacent angles equal to two right angles; therefore *CA* is in a straight line with *AG*.

For the same reason *BA* is also in a straight line with *AH*. And, since the angle *DBC* is equal to the angle *FBA*: for each is right: let the angle *ABC* be added to each; therefore the whole angle *DBA* is equal to the whole angle *FBC*.

And since *DB* is equal to *BC*, and *FB* to *BA*, the two sides *AB*, *BD* are equal to the two sides *FB*, *BC* respectively, and the angle *ABD* is equal to the angle *FBC*; therefore the base *AD* is equal to the base *FC*; and the triangle *ABD* is equal to the triangle *FBC*.

Now the parallelogram *BL* is double of the triangle *ABD*, for they have the same base *BD* and are in the same parallels *BD*, *AL*.

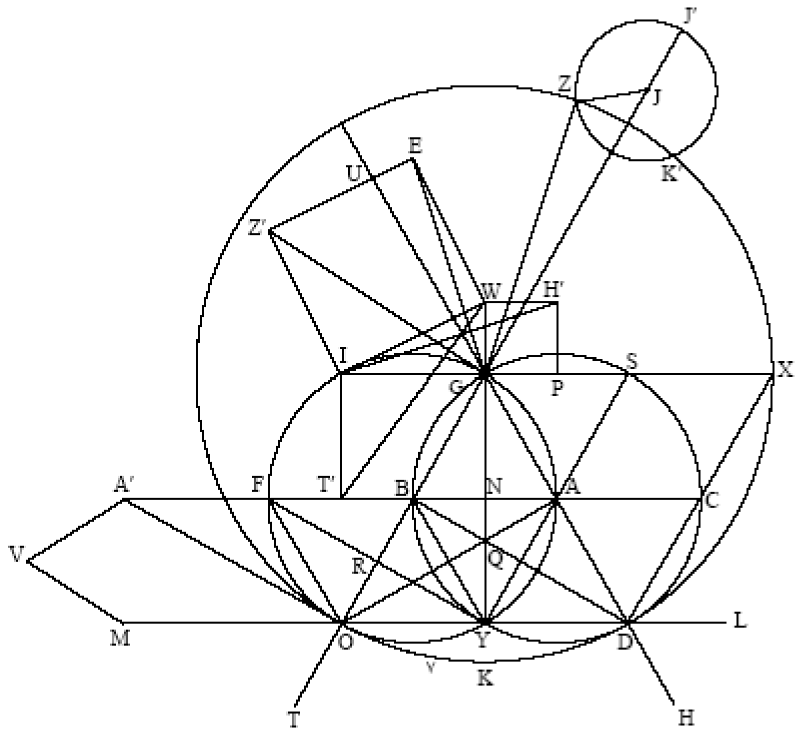
And the square *GB* is double of the triangle *FBC*, for they again have the same base *FB* and are in the same parallels *FB*, *GC*.(but the double of equals are equal to one another).

Therefore the parallelogram *BL* is also equal to the square *GB*.

Similarly if *AE*, *BK* be joined, the parallelogram *CL* can also be proved equal to the same square *HC*; therefore the whole square *BDEC* is equal to the two squares *GB*, *HC*.

And the square *BDEC* is described on *BC*, and the squares *GB*, *HC* on *BS*, *AC*. Therefore the square on the side *BC* is equal to the squares on the sides *BA*, *AC*.

Therefore etc.



3. The connection of theorems with integrated shapes

As it was pointed out, proving each theorem will need the drawing of shapes proper with that. The shapes related to the above mentioned 48 theorems of the First Article of Euclid's Elements are as follows:

The first theorem from two circles of GBY, GA and AGB triangle

The second theorem from DKO circle with one of the two mentioned circles and AGB triangle and line segment of AD or BO

The third theorem from one of the two circles of GBY and Gay and line segment of AD which is detached from AH line segment and BO line segment which is detached from BT line segment

The fourth theorem from two triangles of GDY and GOY

The fifth theorem from two half line of GH and GT and line segments of AB, BE and AO

The sixth theorem from DGO triangle with one of the two line segments of BD or AO

The seventh theorem from quadrilateral of ABOD and its two diameters i.e. BD and AO

The eight theorem from GAB triangle and two triangles of ADO and BDO

The ninth theorem from two half lines of GH and GT and triangle of ABY and ling segment of GY

The tenth theorem from AGB triangle and GN line segment

The eleventh theorem from the line segment of CA' and AGB triangle

The twelfth theorem from DKO circle and half line of ML and GDO triangle and GY line segment

The thirteenth theorem from DO, AY and NY line segments

The fifteenth theorem is proved from two opposite angles to the tope in the shape.

The sixteenth theorem from two triangles of BYO and FYO and half lines of OT and OM

The theorems of seventeen, eighteen, nineteen and twenty will be provable with regard to a few shapes.

The twenty first theorem from the triangle of GDB and GN, BN and NA line segments

The twenty second theorem from two circles of DKO and ZK' and half line of J'T and ling segment of ZG

The twenty third theorem can be proved through some shapes.

The twenty fourth theorem from Quadrilateral of ABOD and ABG triangle

The twenty fifth thermo is proved with regard to different triangles.

The twenty sixth from two triangles of BFO and GDO and two line segments of GY and DB

The twenty seventh, twenty eight and twenty ninth theorems from the line segments of A'C, ML and GT

The thirtieth theorem from line segments of DX, SY, GO and CB

The thirty first, thirty second, thirty third and thirty fourth theorems can be proved with regard to different shapes.

The thirty fifth theorem from the parallelograms of ABDY and FBDY

The thirty sixth theorem from parallelograms of ACDY, FBYO and ABYD

The thirty seventh theorem from parallelogram of BFOY and two triangles of ABY and A'FO

The thirty eight theorem from the parallelogram of ACDY and diameter of AD and parallelogram of BFOY and diameter of BO

The thirty night theorem from ABYD quadrilateral and diameters of AY, BD and line segments of AQ and QY

The fortieth theorem from triangles of CDA and FBO and line segments of CF, DO, DR and FR

The forty first theorem from the parallelogram of ABOY and triangle of FYO and two line segments of AF and AO

The forty second theorem from the triangle of GDO and parallelogram of SXDY and line segment of GY

The forty third theorem from GXDO parallelogram and diameter of DG and line segments of CB and SY

The forty fourth theorem from GXDO parallelogram and BOF triangle and angle of FOA'

The theorems of forty five and forty six are proved with regard to different shapes

The theorem of forty seventh from triangle of GIW and three squares of GPH'W, GIT'N and EZ'IW and line segments of GU, GZ', GE, T'W and IH'.

The theorem forty eights from the triangle of AGB and perpendicular of GN

Conclusion

The integrated shape drawn by Qotboldin Shirazi is in fact a compact diskette which presented approximately the generalities of the theorems of the first articles. He believes that this method is applicable to articles too. Thus, he might be considered as one of the pioneers of compacting the courses. This method in particular in geometry will orient students of high schools to learn how it is possible to understand the simple issues hidden in that shape by analyzing a complex shape on one hand and to summary the materials by combining the subjects of an article, a chapter or a part.

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